How to Construct an Ideal Cipher from a Small Set of Public Permutations

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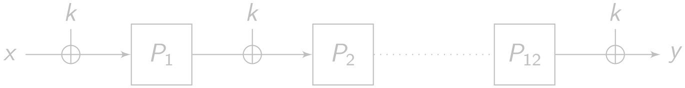
Summary

- We show how to construct an ideal cipher from a small set of n-bit public random permutations $\{P_1, \ldots, P_r\}$
- The construction we consider is the single-key iterated Even-Mansour cipher (*aka* key-alternating cipher) with 12 rounds:



$\Rightarrow$ this yields a family of 2^n permutations indexed by the n-bit key k from only 12 public n-bit permutations

- We show that this construction "behaves" as an ideal cipher with n-bit blocks and n-bit keys using the indifferentiability framework
- We also show that at least 4 rounds are necessary to achieve indifferentiability from an ideal cipher

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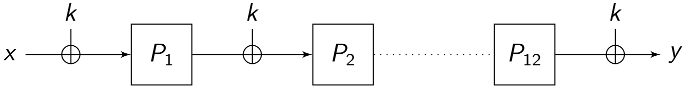

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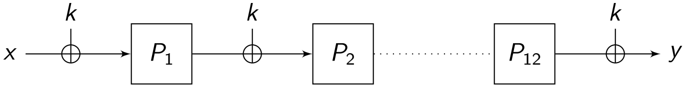

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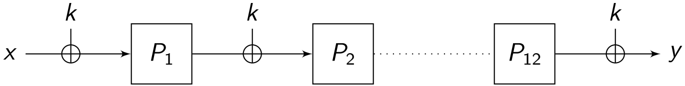

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1. Background on the Iterated Even-Mansour Cipher
2. Indifferentiability of the IEM cipher
   - Formalizing the problem
   - Which key schedule?
   - At least 4 rounds are necessary
3. Indifferentiability proof for 12 rounds

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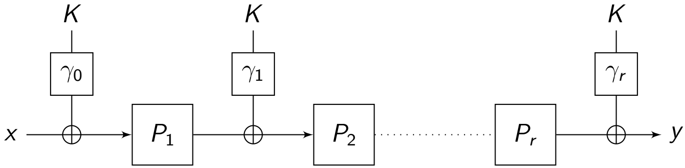
Iterated Even-Mansour cipher (*aka* key-alternating cipher)

Iterated Even-Mansour (IEM) with r rounds:



- The P_i's are public permutations on $\{0,1\}^n$
- $K \in \{0,1\}^\ell$ is the (master) key
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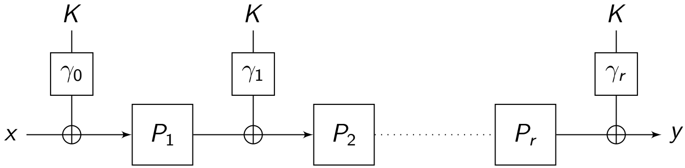

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Most (if not all) SPN ciphers can be described as key-alternating ciphers. E.g. for AES-128, one has $r = 10$, the γ_i's are efficiently invertible permutations, and:

$$P_1 = \ldots = P_9 = \mathtt{SubBytes} \circ \mathtt{ShiftRows} \circ \mathtt{MixColumns}$$
$$P_{10} = \mathtt{SubBytes} \circ \mathtt{ShiftRows}$$

When the P_i's are fixed permutations, one can prove results like:

- the best differential characteristic over $r' < r$ rounds has probability at most p
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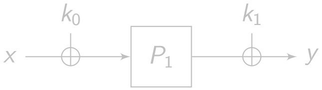
Analysis in the Random Permutation Model (RPM)

Recently, a lot of results have been obtained in the Random Permutation Model: the P_i's are viewed as oracles to which the adversary can make black-box queries (both to P_i and P_i^{-1}).

Interpretation: gives a guarantee against any adversary which does not use particular properties of the P_i's

In fact, this model was already considered 15 years ago by Even and Mansour for $r = 1$ round: they showed that the following cipher is pseudorandom up to $\mathcal{O}(2^{n/2})$ queries of the adversary, when P_1 is a public random permutation:



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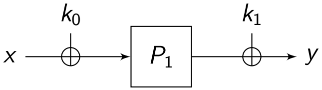

Pseudorandomness of the IEM cipher (in the RPM)

The following results have been successively obtained for the pseudorandomness of the IEM cipher (notation: $N = 2^n$):

- for $r = 1$ round, security up to $\mathcal{O}(N^{\frac{1}{2}})$ queries [EM97]
- for $r \geq 2$, security up to $\mathcal{O}(N^{\frac{2}{3}})$ queries [BKL$^+$12]
- for $r \geq 3$, security up to $\mathcal{O}(N^{\frac{3}{4}})$ queries [Ste13]
- for any even r, security up to $\mathcal{O}(N^{\frac{r}{r+2}})$ queries [LPS12]
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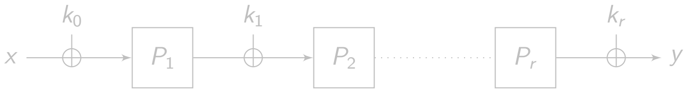

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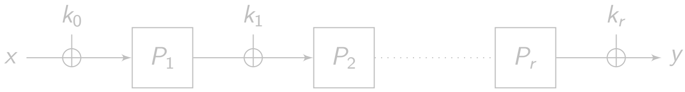

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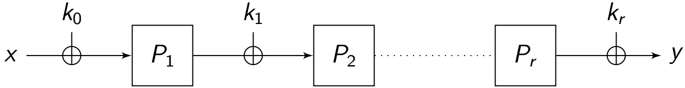

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From indistinguishability to indifferentiability

Previous results state that the IEM cipher is a (strong) pseudorandom permutation (in the random permutation model)
= usual single, secret-key security model

Question

What about related-, known- or chosen-key attacks?
Can we even hope to prove that the IEM "behaves" as (*is indifferentiable from*) an ideal cipher?

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A word on the ideal cipher model

- the pseudorandomness security notion for a block cipher is sufficient to prove the security of a lot of applications (encryption modes and MACs)
- however, sometimes it is not sufficient (e.g. for block cipher-based hash functions like Davies-Meyer mode)
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- similar to the random oracle model for a hash function
- warning: instantiation problems as well (no concrete block cipher can be proved to be an ideal cipher in any reasonable sense)
- though we cannot prove that a block cipher behaves as an ideal cipher in the standard model, we can prove results in idealized models (e.g. the Random Permutation Model in the case of the IEM cipher)
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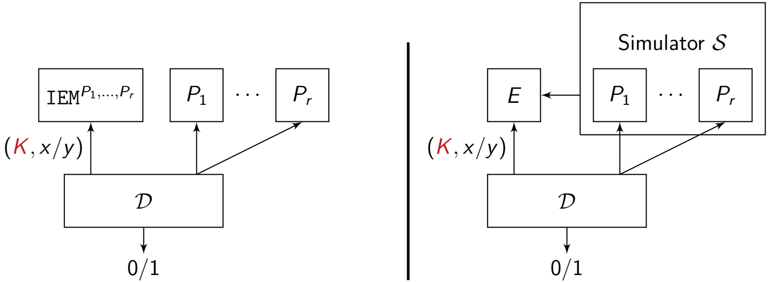
Indifferentiability: definition

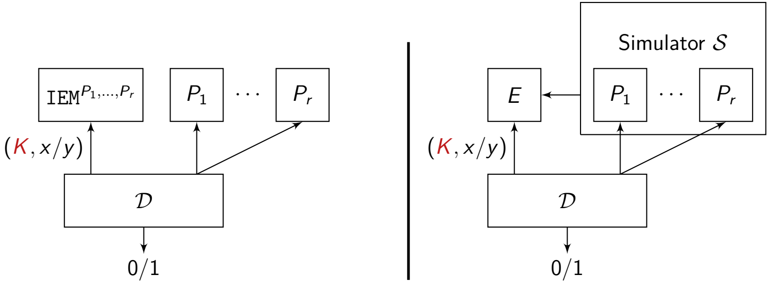
Definition

The IEM cipher $\mathtt{IEM}^{P_1,\ldots,P_r}$ with random permutations $\boldsymbol{P} = (P_1, \ldots, P_r)$ is said indifferentiable from an ideal cipher E if there exists a polynomial time simulator $\mathcal{S}$ with oracle access to E such that the two systems $(\mathtt{IEM}^{\boldsymbol{P}}, \boldsymbol{P})$ and $(E, \mathcal{S}^E)$ are indistinguishable.



Indifferentiability: definition

NB: The distinguisher specifies the plaintext/ciphertext and the key when querying $\mathtt{IEM}^{P_1,\ldots,P_r}$ or E.



The answers of the simulator $\mathcal{S}$ must be:

- coherent with answers the distinguisher can obtain directly from E
- close in distribution to the answers of random permutations

Composition theorem

Usefulness of indifferentiability: composition theorem

Theorem

If a cryptosystem Γ is secure when used with an ideal cipher E, and if $\mathtt{IEM}^{P_1,\ldots,P_r}$ (for sufficiently many rounds) is indifferentiable from E, then Γ is also secure when used with $\mathtt{IEM}^{P_1,\ldots,P_r}$ with random permutations $P_1, \ldots, P_r$ (for single-stage security notions).

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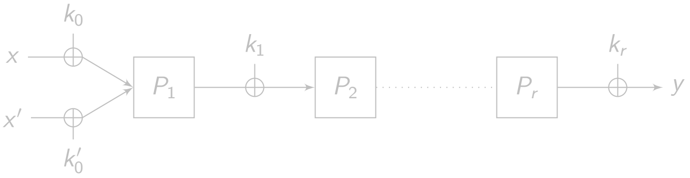

IEM with independent round keys is not indifferentiable from an ideal cipher with key space $\{0,1\}^{(r+1)n}$ because of the following distinguisher:

- choose an arbitrary $x \in \{0,1\}^n$ and $k_0 \in \{0,1\}^n$
- define $x' = x \oplus c$ and $k'_0 = k_0 \oplus c$ with c a non-zero constant
- let $K = (k_0, k_1, \ldots, k_r)$ and $K' = (k'_0, k_1, \ldots, k_r)$
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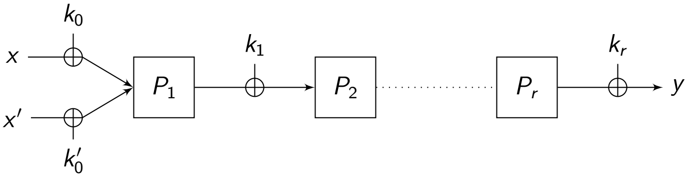

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Proving indifferentiability for the IEM cipher

Independent keys leave too much “freedom” to the adversary.

Two ideas to solve the problem:

1. add a key schedule, and put some cryptographic assumption on it
   $\Rightarrow$ Andreeva et al. CRYPTO 2013 [ABD$^+$13]
2. restrain the key space and correlate the round keys, e.g. $(k, k, \ldots, k)$
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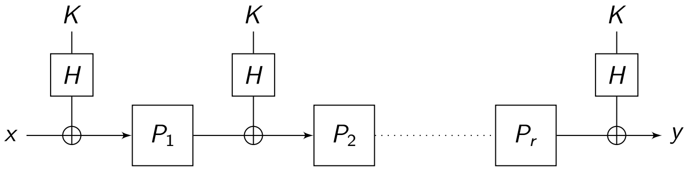

$\rightarrow$ indifferentiable from an ideal cipher with ℓ-bit keys for $r = 5$ ([ABD$^+$13] gives attacks up to 3 rounds)

Better bounds and less rounds than in this paper.

But the assumption about the key derivation is very strong and far from concrete designs (the key-schedule is often invertible)

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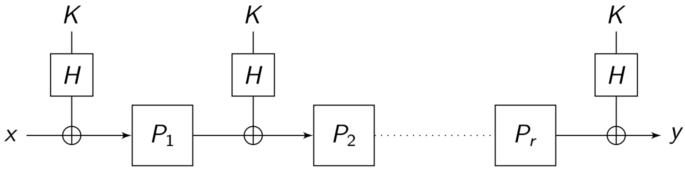

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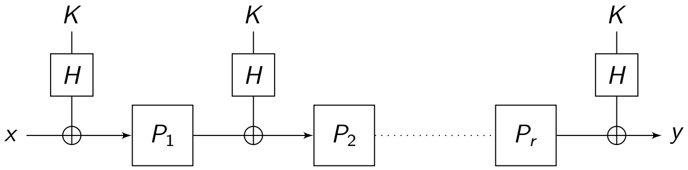

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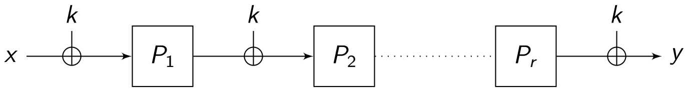

The trivial attack on independent keys does not apply → is it indiff. from an ideal cipher for sufficiently many rounds ?

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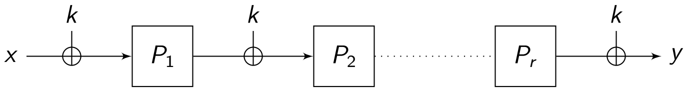

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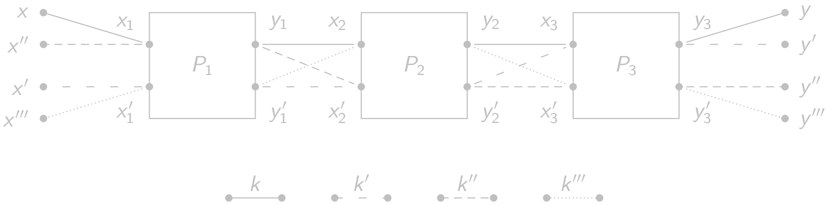

One can (easily) find (x, x', x'', x'''), (y, y', y'', y''') and (k, k', k'', k''') such that $y = \mathtt{IEM}^{(P_1,P_2,P_3)}(k, x)$, etc. and:

$$\begin{cases} k \oplus k' \oplus k'' \oplus k''' = 0 \\ x \oplus x' \oplus x'' \oplus x''' = 0 \\ y \oplus y' \oplus y'' \oplus y''' = 0 \end{cases} .$$

Finding such values can be showed to be hard for an ideal cipher.

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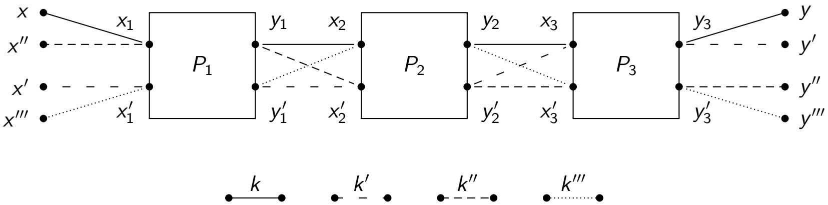

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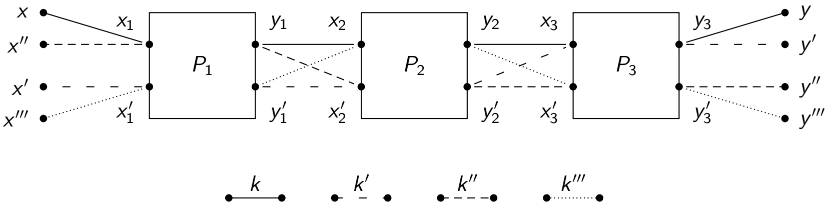

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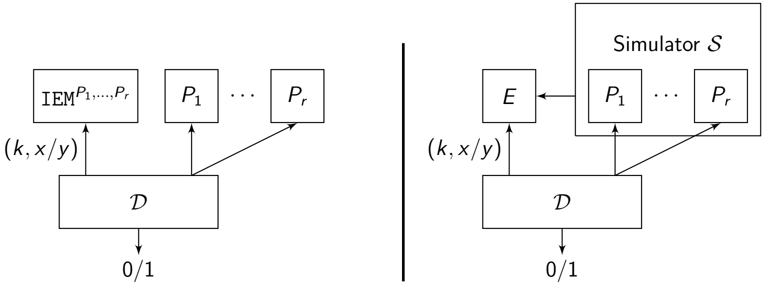
Reminder: the indifferentiability setting



Simulation: general strategy

The simulator must return answers that are coherent with what the distinguisher can obtain from the ideal cipher E, i.e.:

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For this, the simulator must adapt at least one permutation to "match" what is given by the ideal cipher.

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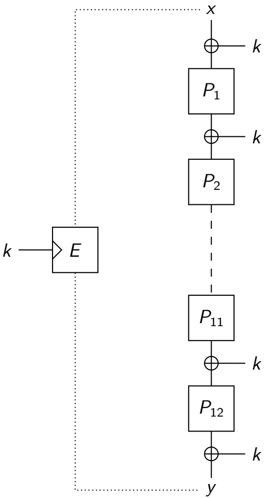

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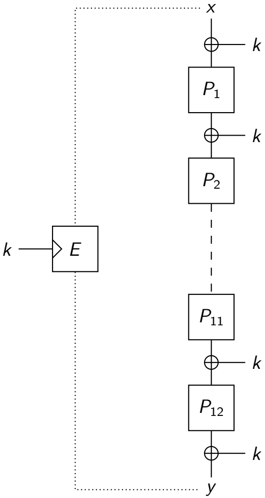

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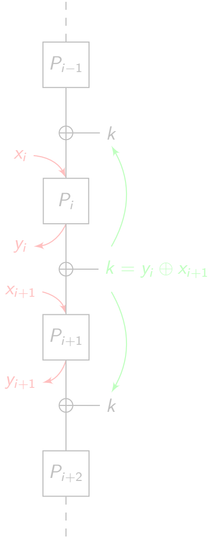

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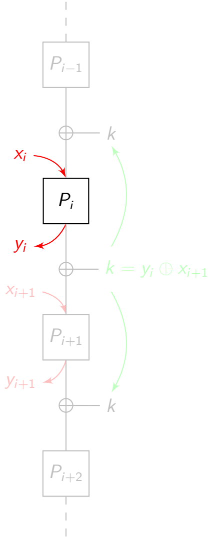

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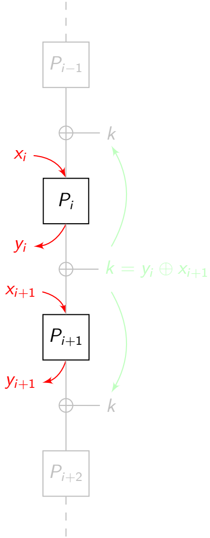

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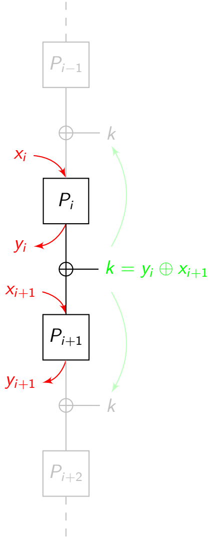

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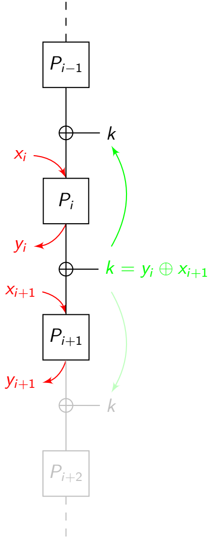

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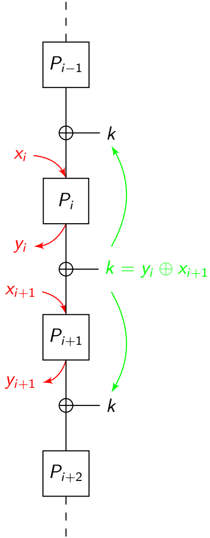

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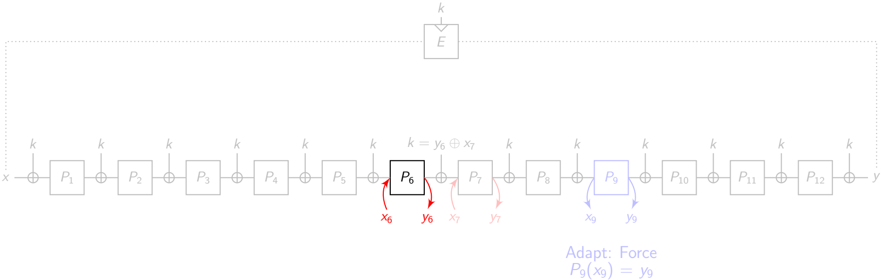

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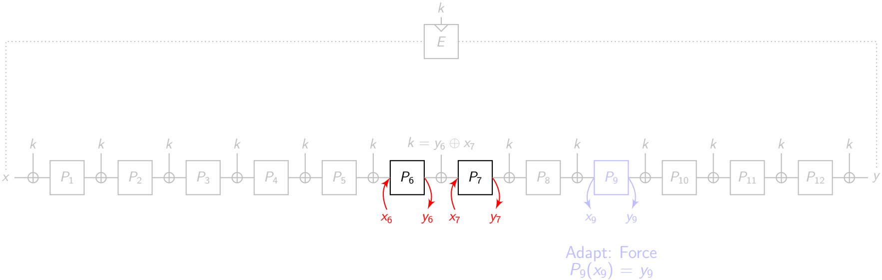

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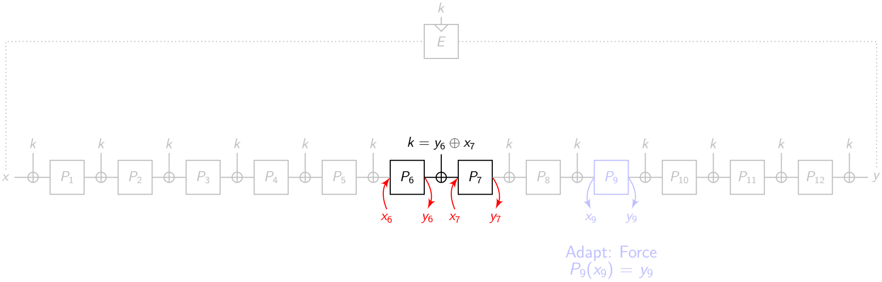

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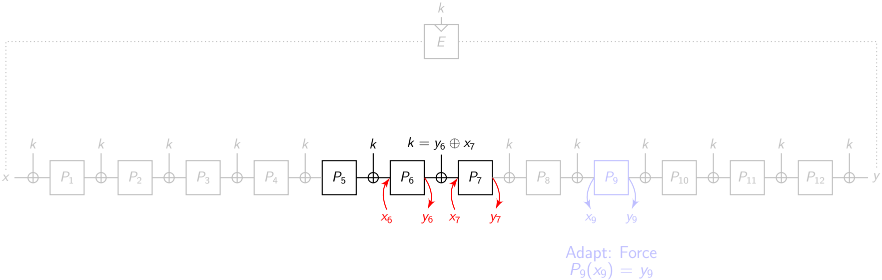

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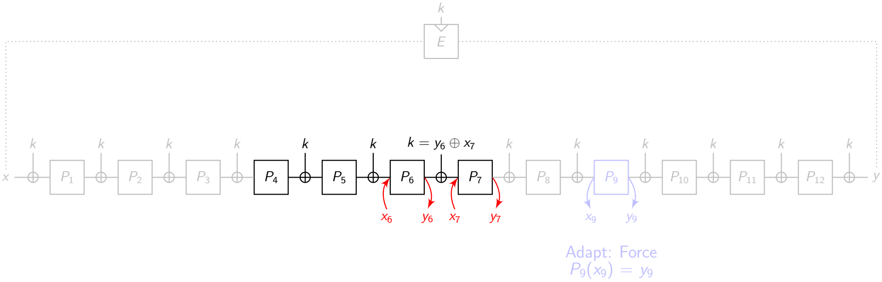

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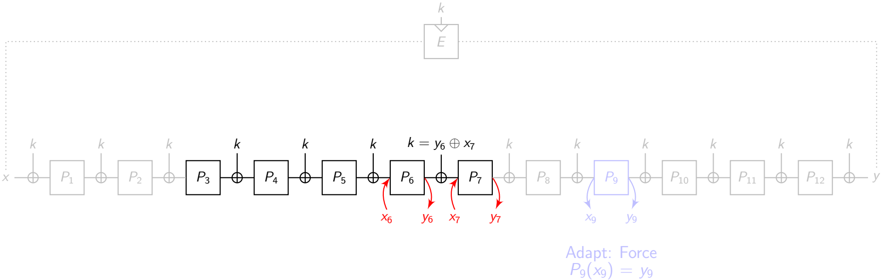

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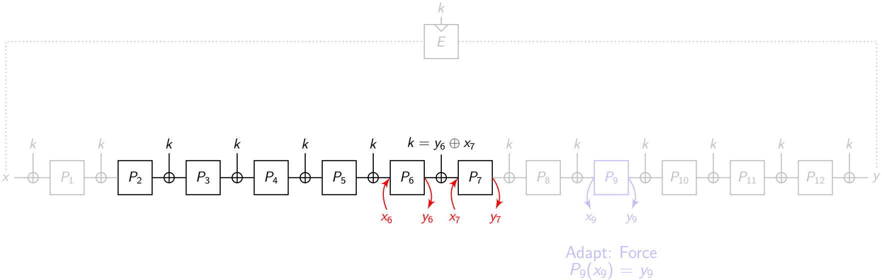

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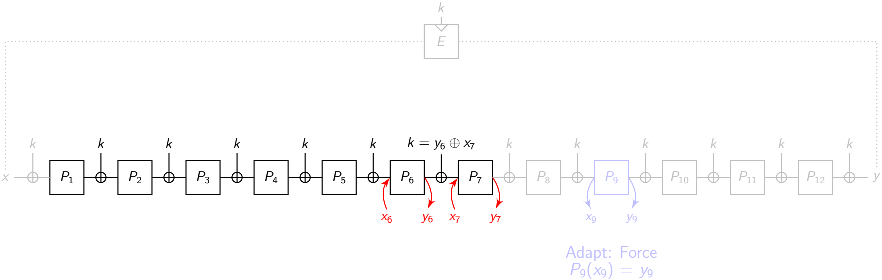

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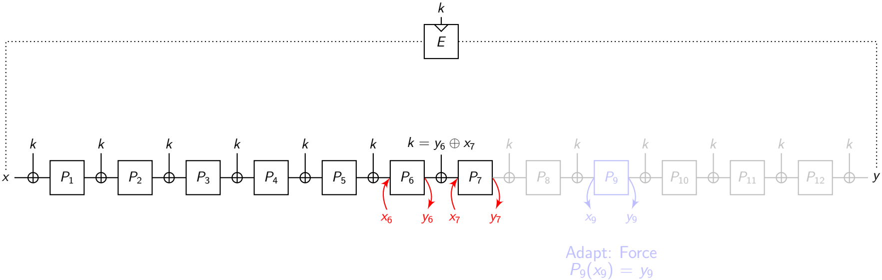

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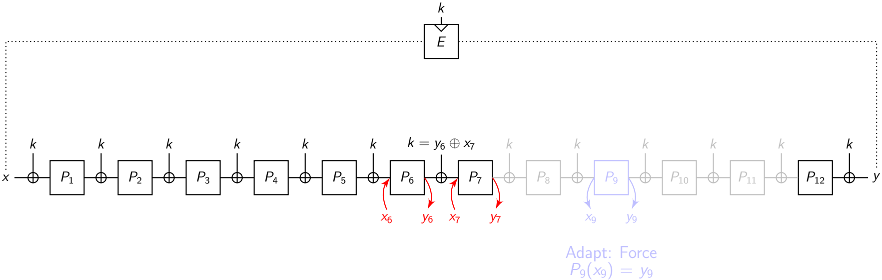

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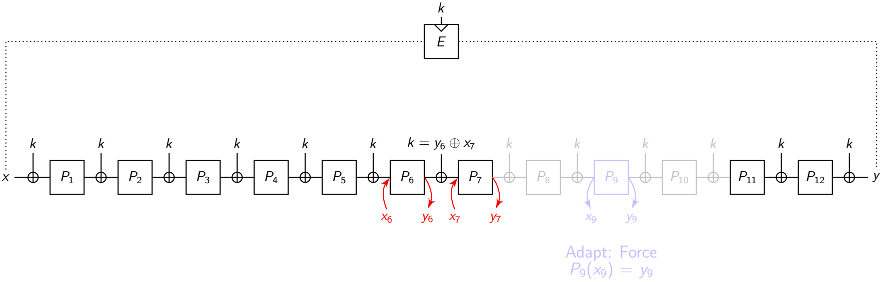

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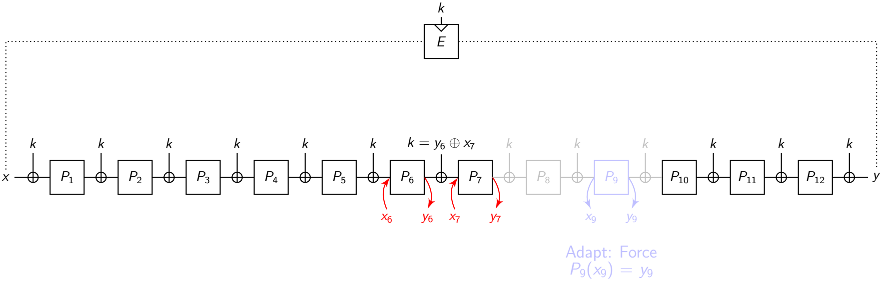

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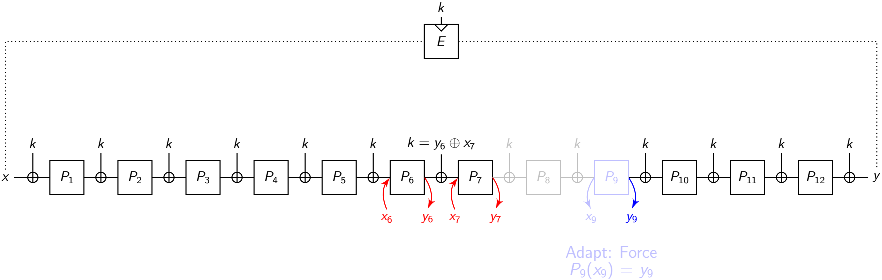

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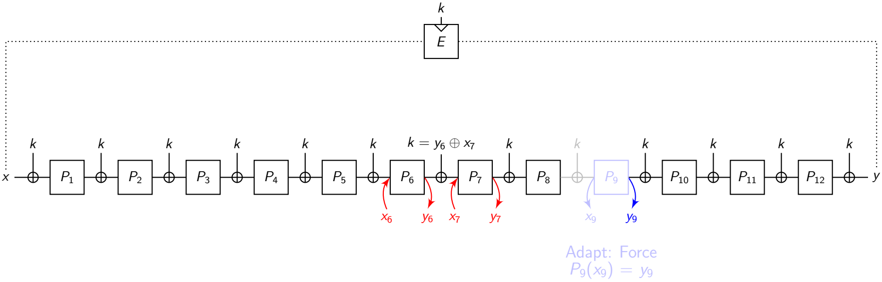

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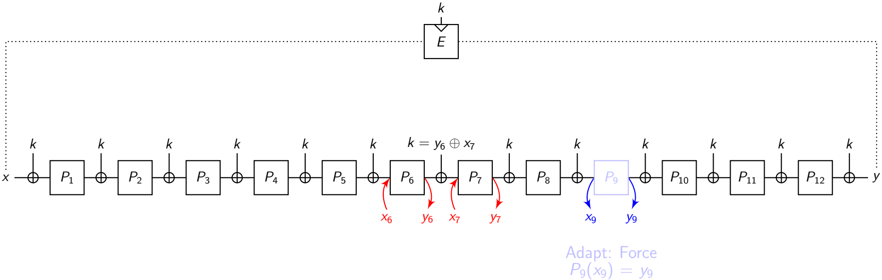

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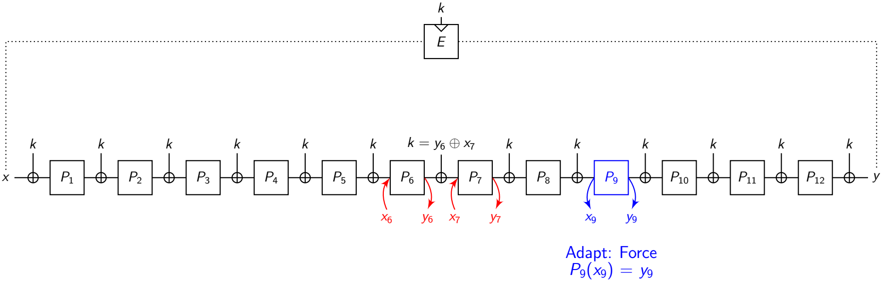

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What could go wrong during simulation

Two problems to deal with:

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   - completing a partial chain creates new chains, which must be completed, creating new partial chains, etc.
   - ⇒ potential blow-up of the number of chains completed by the simulator
   - but the simulator must be polynomial-time!
2. impossibility to adapt:
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- the simulator only detects and completes partial chains at very specific places:
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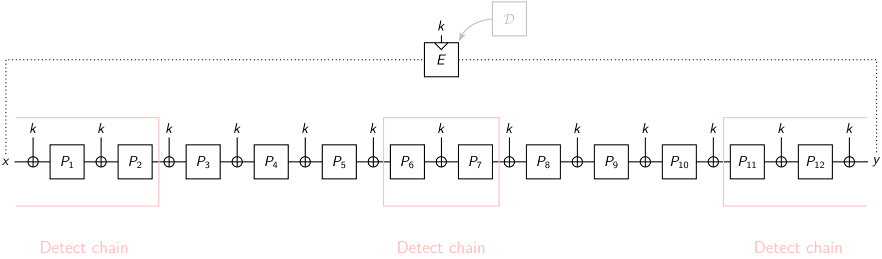

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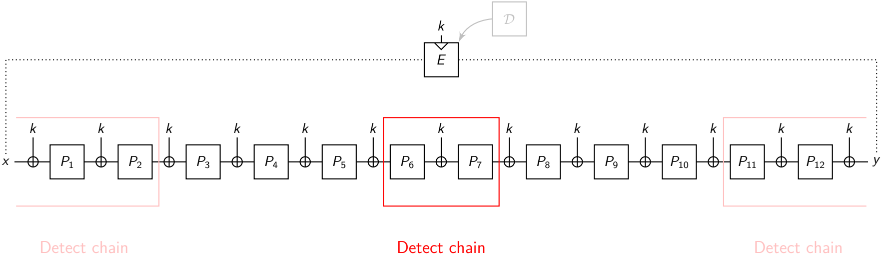

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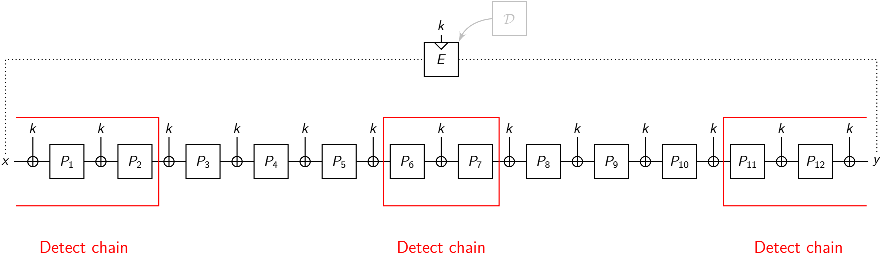

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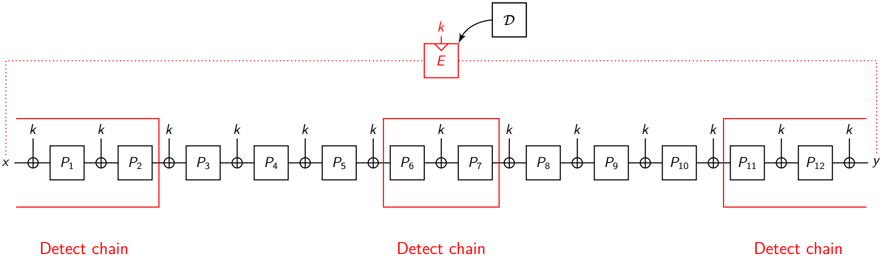

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- chains are always adapted at P_4 or P_9
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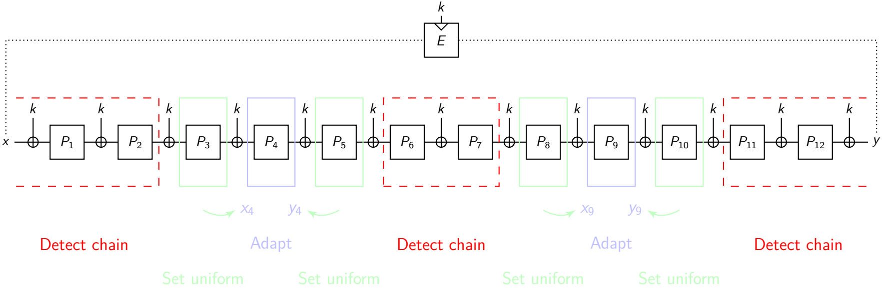

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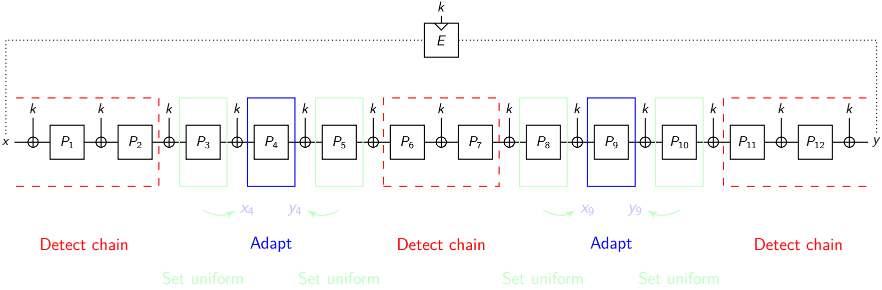

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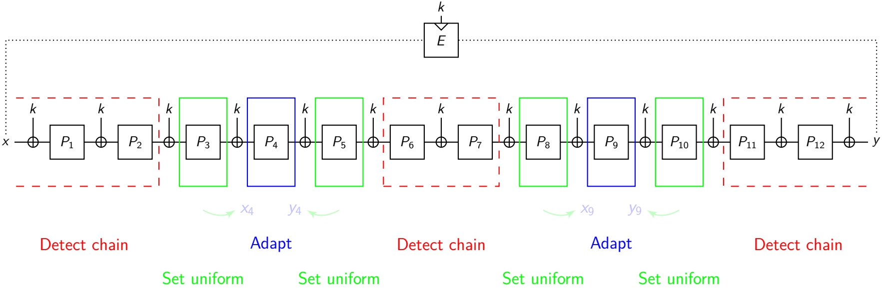

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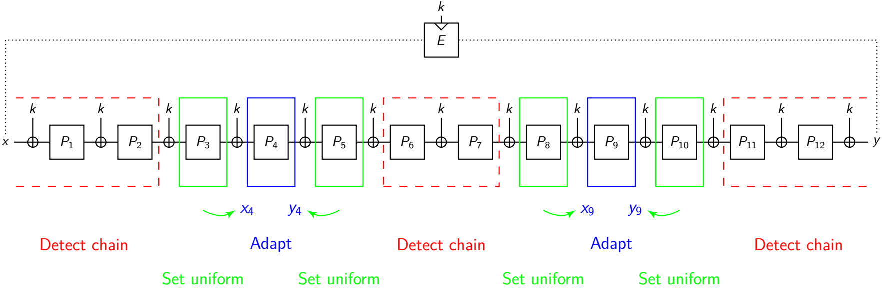

Conclusion

Main result

The single-key IEM cipher with 12 rounds is indifferentiable from an ideal cipher with n-bit keys.

Interpretation of the result:

- shows that the general strategy of building block ciphers from SPNs is sound and may even yield something close to an ideal cipher
- says little about concrete block ciphers: e.g. the permutations P_1, $\ldots$, P_{10} of AES-128 are too simple and not independent
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Open problems

1. exact number of rounds for indifferentiability?
   - The indifferentiability proof requires 12 rounds...
     but the best attack is only on 3 rounds.

Conjecture

The single-key IEM with $3 < r < 12$ rounds is indifferentiable from an ideal cipher with n-bit keys

   - $r = 4$ may well be sufficient
     (we explain which obstacles appear already for $r = 8$ in the full paper)

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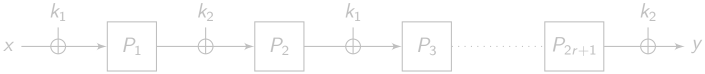

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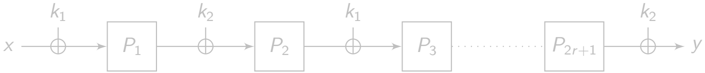

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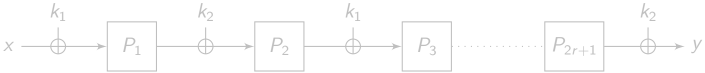

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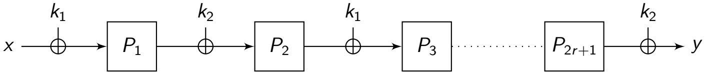
     (we explain which obstacles appear already for $r = 8$ in the full paper)

2. construction with $2n$-bit keys? (or more generally tn-bit keys with $t > 1$)



The end. . .

Thanks for your attention!
Comments or questions?

References I

Elena Andreeva, Andrey Bogdanov, Yevgeniy Dodis, Bart Mennink, and John P. Steinberger.

On the Indifferentiability of Key-Alternating Ciphers.

In Ran Canetti and Juan A. Garay, editors, *Advances in Cryptology - CRYPTO 2013 (Proceedings, Part I)*, volume 8042 of *Lecture Notes in Computer Science*, pages 531–550. Springer, 2013.

Full version available at `http://eprint.iacr.org/2013/061`.

Andrey Bogdanov, Lars R. Knudsen, Gregor Leander, François-Xavier Standaert, John P. Steinberger, and Elmar Tischhauser.

Key-Alternating Ciphers in a Provable Setting: Encryption Using a Small Number of Public Permutations - (Extended Abstract).

In David Pointcheval and Thomas Johansson, editors, *Advances in Cryptology - EUROCRYPT 2012*, volume 7237 of *Lecture Notes in Computer Science*, pages 45–62. Springer, 2012.

References II

Jean-Sébastien Coron, Jacques Patarin, and Yannick Seurin.
The Random Oracle Model and the Ideal Cipher Model Are Equivalent.
In David Wagner, editor, *Advances in Cryptology - CRYPTO 2008*, volume 5157 of *Lecture Notes in Computer Science*, pages 1–20. Springer, 2008.

Shan Chen and John Steinberger.
Tight Security Bounds for Key-Alternating Ciphers.
IACR Cryptology ePrint Archive, Report 2013/222, 2013.
Available at `http://eprint.iacr.org/2013/222`.

Shimon Even and Yishay Mansour.
A Construction of a Cipher from a Single Pseudorandom Permutation.
Journal of Cryptology, 10(3):151–162, 1997.

References III

Thomas Holenstein, Robin Künzler, and Stefano Tessaro.

The Equivalence of the Random Oracle Model and the Ideal Cipher Model, Revisited.

In Lance Fortnow and Salil P. Vadhan, editors, *Symposium on Theory of Computing - STOC 2011*, pages 89–98. ACM, 2011.

Full version available at `http://arxiv.org/abs/1011.1264`.

Rodolphe Lampe, Jacques Patarin, and Yannick Seurin.

An Asymptotically Tight Security Analysis of the Iterated Even-Mansour Cipher.

In Xiaoyun Wang and Kazue Sako, editors, *Advances in Cryptology - ASIACRYPT 2012*, volume 7658 of *Lecture Notes in Computer Science*, pages 278–295. Springer, 2012.

References IV

Ueli M. Maurer, Renato Renner, and Clemens Holenstein.

Indifferentiability, Impossibility Results on Reductions, and Applications to the Random Oracle Methodology.

In Moni Naor, editor, *Theory of Cryptography Conference- TCC 2004*, volume 2951 of *Lecture Notes in Computer Science*, pages 21–39. Springer, 2004.

John Steinberger.

Counting solutions to additive equations in random sets.

arXiv Report 1309.5582, 2013.

Available at http://arxiv.org/abs/1309.5582.