Security Analysis of Key-Alternating Ciphers in the Even-Mansour Model

Yannick Seurin

ANSSI

March 20, 2015 — CCA Seminar

The Title

Security Analysis

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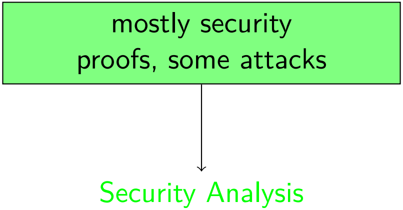

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Security Analysis of Key-Alternating Ciphers

in the Even-Mansour Model

a generic model where some internal components of BCs are modeled as public random oracles

Talk Mainly Based on Joint Work with:

- Jacques Patarin (Versailles Univ.)
- Rodolphe Lampe (Versailles Univ.)
- Benoît Cogliati (Versailles Univ.)
- Jooyoung Lee (Sejong Univ.)
- John Steinberger (Tsinghua Univ.)
- Shan Chen (Tsinghua Univ.)

Outline

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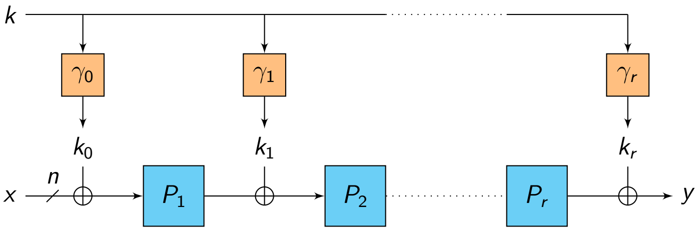
Key-Alternating Cipher (KAC): Definition



An r-round key-alternating cipher:

- plaintext $x \in \{0,1\}^n$, ciphertext $y \in \{0,1\}^n$
- master key $k \in \{0,1\}^\kappa$
- the P_i's are public permutations on $\{0,1\}^n$
- the γ_i's are key derivation functions mapping k to n-bit "round keys"
- examples: most SPNs (AES, SERPENT, PRESENT, LED, ...)

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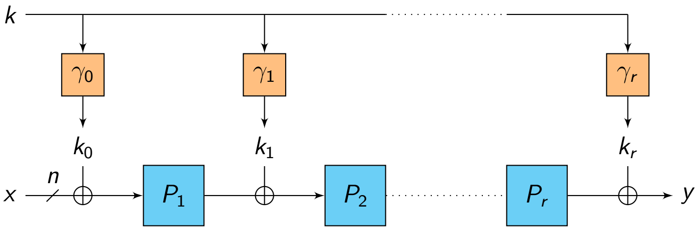

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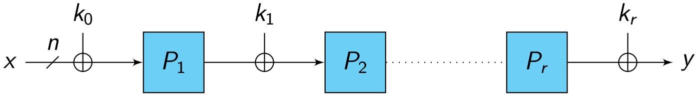

Round keys can be:

- independent (total key-length $\kappa = (r+1)n$)
- derived from an n-bit master key ($\kappa = n$), e.g.
  - trivial key-schedule: $(k, k, \ldots, k)$
  - more complex: $(\gamma_0(k), \gamma_1(k), \ldots, \gamma_r(k))$
- anything else (e.g. $2n$-bit master key (k_0, k_1) and round keys $(k_0, k_1, k_0, k_1, \ldots)$ as in LED-128)

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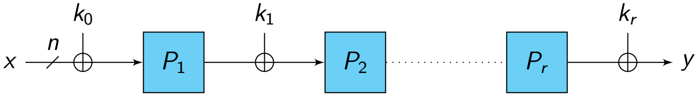

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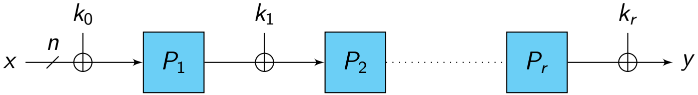

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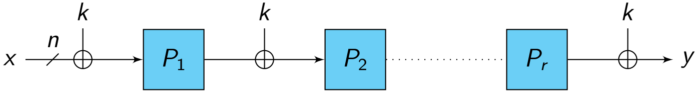

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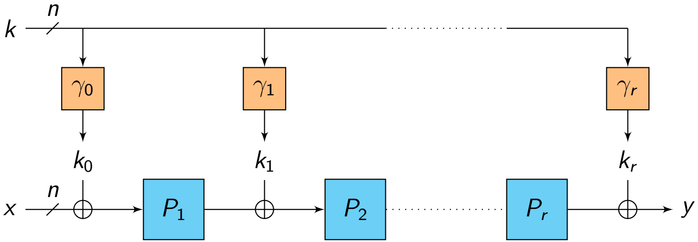

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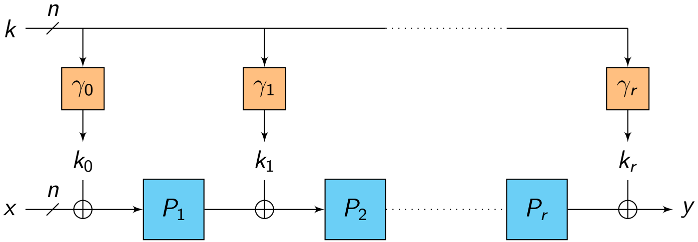

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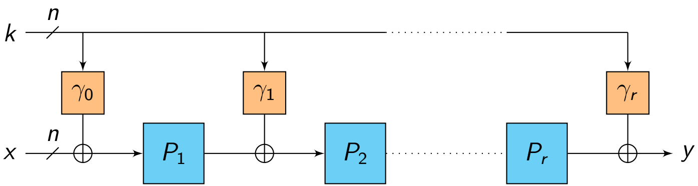

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How can we "prove" security?

- against a general adversary:
  $\Rightarrow$ too hard (unconditional complexity lower bound!)
- against specific attacks (differential, linear...):
  $\Rightarrow$ use specific design of $P_1, \ldots, P_r$ (count active S-boxes, etc.)
- against generic attacks:
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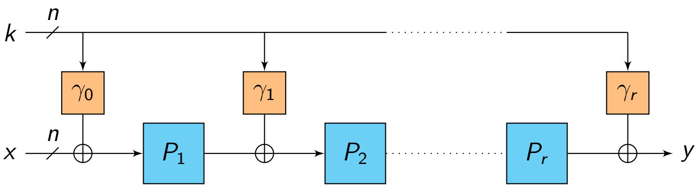

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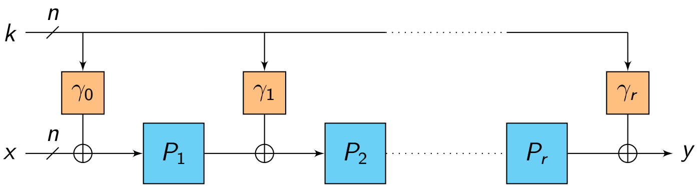

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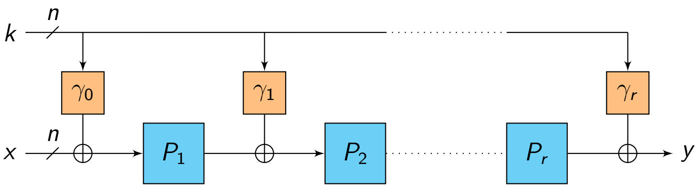

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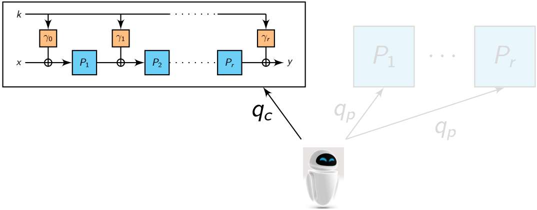
Analyzing KACs in the Random Permutation Model



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- complexity measure of the adversary:
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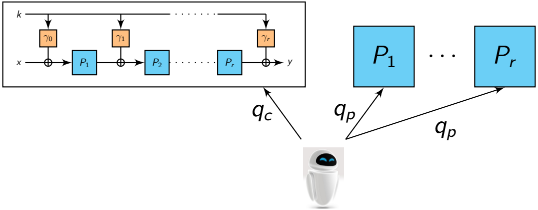

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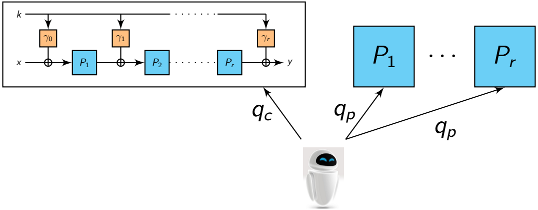

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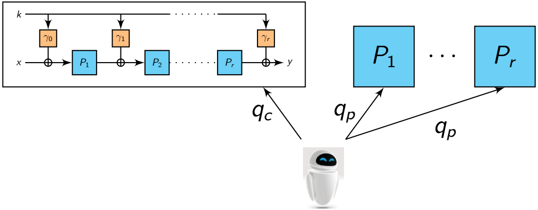

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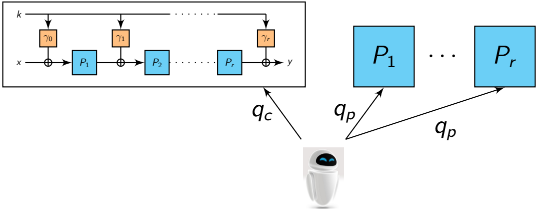

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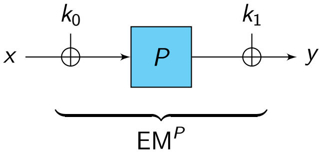

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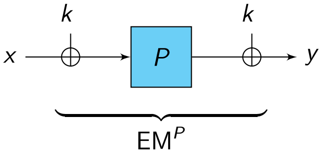

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20 Years After

Revival of the approach:

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- EUROCRYPT 2012 paper by Bogdanov *et al.*
- they showed that for $r = 2$ the security bound is pushed back to $\mathcal{O}(2^{\frac{2n}{3}})$ adversarial queries
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  (at least 10 proof papers + 6 cryptanalysis papers)
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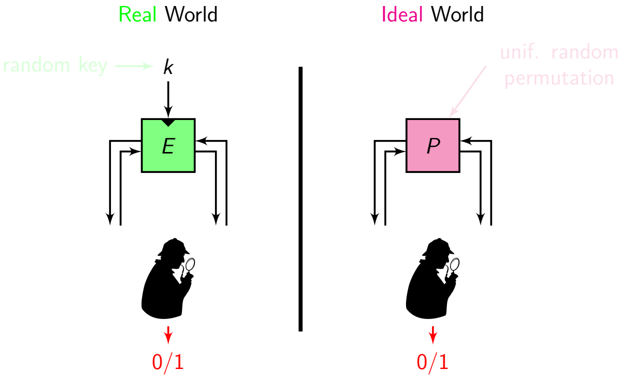
Formalizing Block Cipher Security: Pseudorandomness



SPRP (*a.k.a.* CCA) advantage:

$$\mathbf{Adv}_E^{\mathrm{sprp}}(\mathcal{D}) = \left| \Pr\left[\mathcal{D}^{E_k} = 1\right] - \Pr\left[\mathcal{D}^{P} = 1\right] \right|$$

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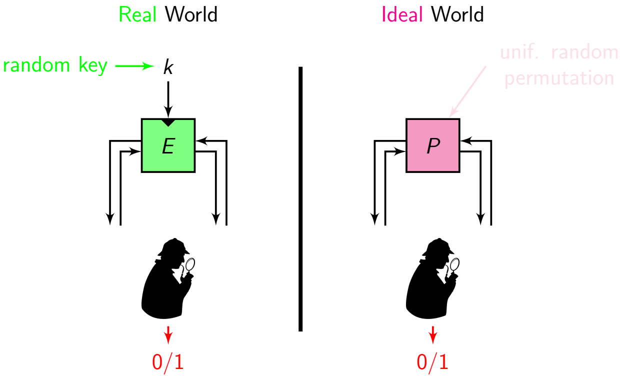

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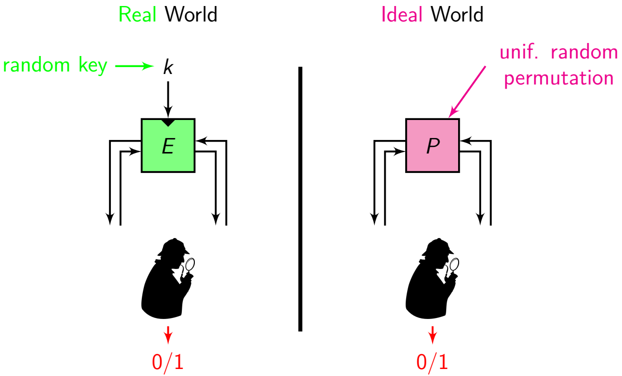

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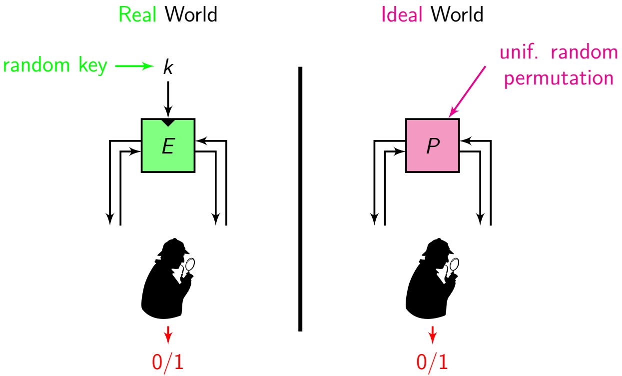

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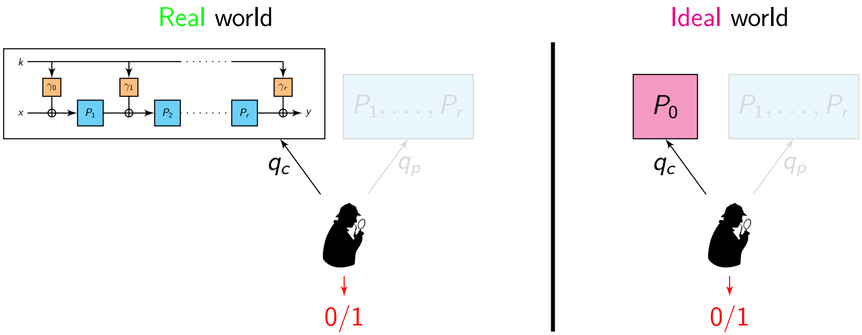
Formalizing Pseudorandomness for the IEM Cipher



- real world: IEM cipher with a random key $k \leftarrow_{\$} \{0,1\}^{\kappa}$
- ideal world: random permutation P_0 independent from $P_1, \ldots, P_r$
- Rand. Perm. Model: $\mathcal{D}$ has oracle access to $P_1, \ldots, P_r$ in both worlds
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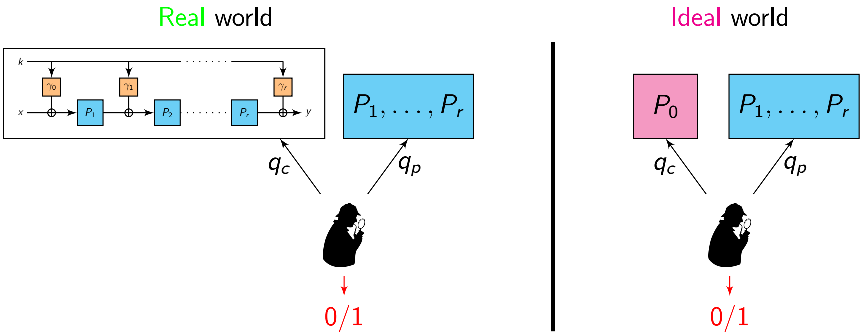

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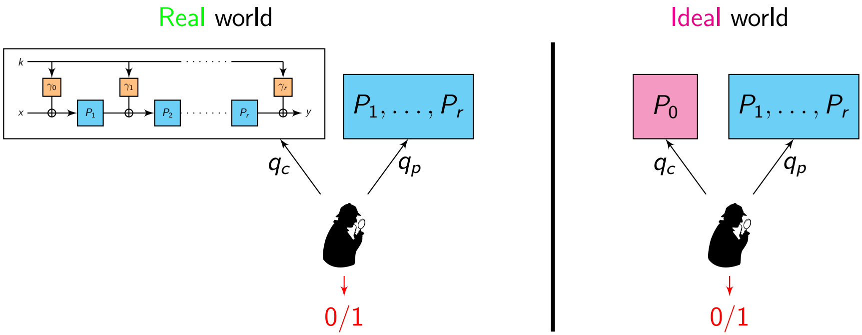

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Pseudorandomness of the IEM Cipher: Main Result

Theorem (Chen-Steinberger [CS14])

For independent round keys $(k_0, \ldots, k_r)$ and independent inner permutations $P_1, \ldots, P_r$, the best distinguishing advantage against the r-round IEM cipher satisfies

$$\mathbf{Adv}^{\mathrm{sprp}}_{\mathsf{EM}[n,r]}(q_c, q_p) \leq \mathcal{O}\left(\frac{q_c q_p^r}{2^{rn}}\right)$$

- holds when the $r+1$ round keys are only r-wise independent, e.g.

$$(k_0, k_0 \oplus k_1, k_1 \oplus k_2, \ldots, k_{r-1} \oplus k_r, k_r)$$

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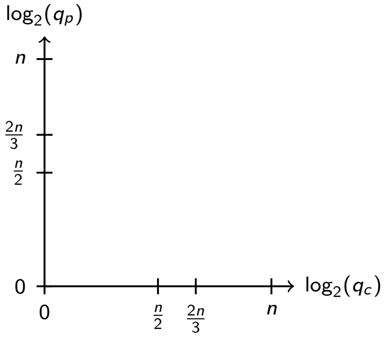

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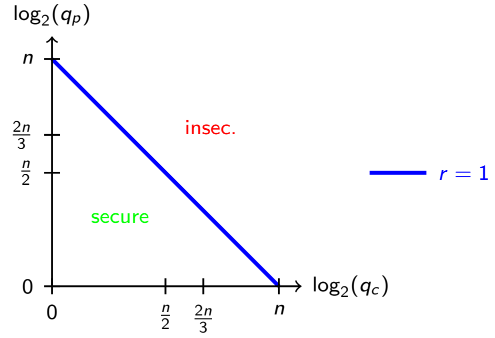

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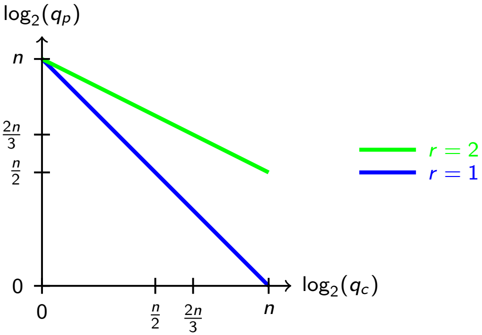

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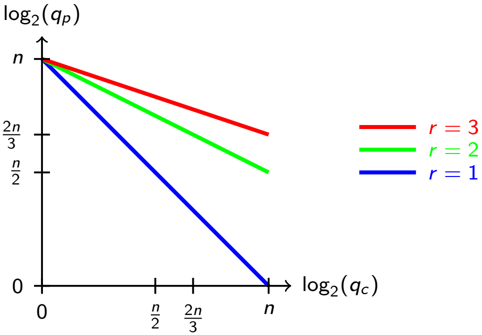

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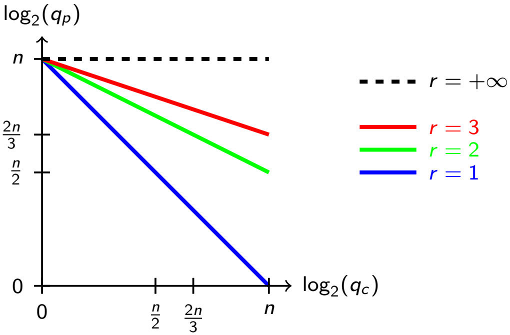

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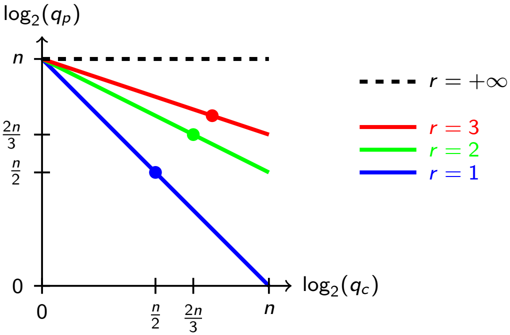

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Is it possible to prove a similar $\mathcal{O}(2^{\frac{rn}{r+1}})$ bound when:

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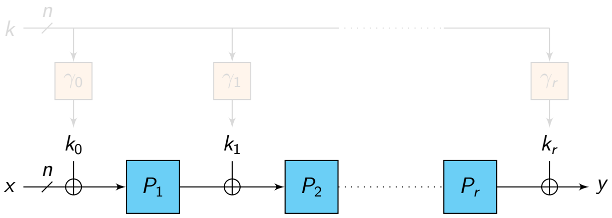
- and/or when the same permutation P is used at each round

as is the case in many concrete designs (AES, etc.)?



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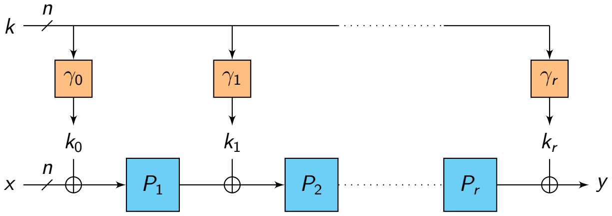

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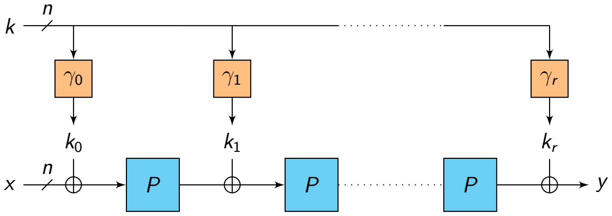

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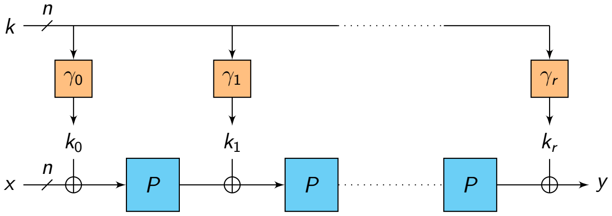

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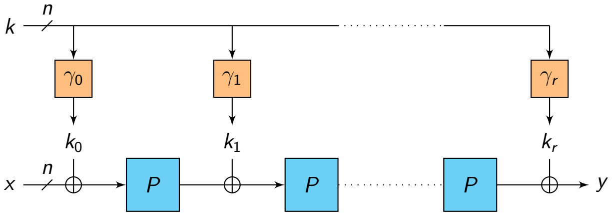

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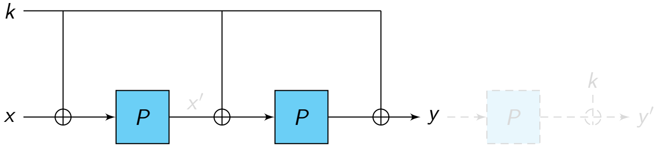

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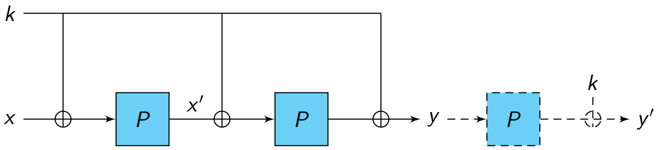

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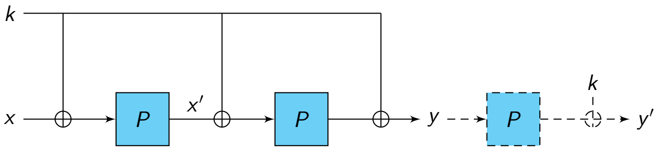

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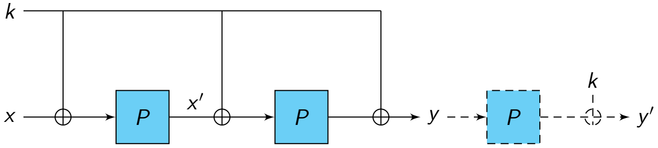

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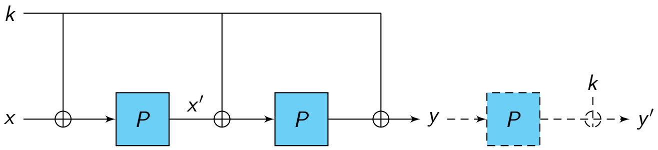

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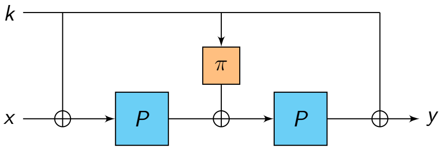
Provably Secure Construction for 2 Rounds

Theorem (Chen *et al.* [CLL+14])

The IEM cipher below is secure up to $\widetilde{\mathcal{O}}(2^{\frac{2n}{3}})$ queries of the adversary.



π can be any fixed ($\mathbb{F}_2$-linear) orthomorphism (i.e., π is a permutation and $k \mapsto k \oplus \pi(k)$ is a permutation), for instance

$$\pi : (k_L, k_R) \mapsto (k_R, k_L \oplus k_R) \quad \text{(Feistel)}$$
$$\pi : k \mapsto c \odot k, \quad \text{for } c \neq 0, 1 \quad \text{(field mult.)}$$

Outline

Introduction and History

Pseudorandomness of Key-Alternating Ciphers

Beyond Pseudorandomness: Related-Key Attacks

Beyond RKAs: Chosen-Key Attacks and Indifferentiability

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- stronger adversarial model: the adversary can specify Related-Key Deriving (RKD) functions ϕ and receive $E_{\phi(k)}(x)$ and/or $E^{-1}_{\phi(k)}(y)$
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$$\{\phi_\Delta : k \mapsto k \oplus \Delta, \Delta \in \{0,1\}^\kappa\}$$

- NB: independent work by Farshim and Procter at FSE 2015 [FP15]

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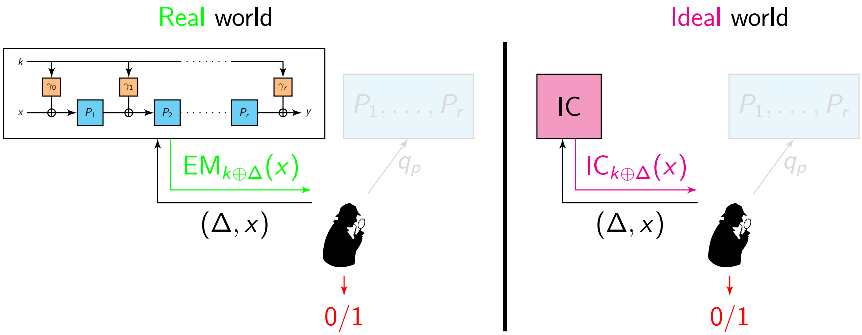
XOR-RKAs against the IEM Cipher: Formalization



- real world: IEM cipher with a random key $k \leftarrow_{\$} \{0,1\}^{\kappa}$
- ideal world: ideal cipher IC independent from $P_1, \ldots, P_r$
- Rand. Perm. Model: $\mathcal{D}$ has oracle access to $P_1, \ldots, P_r$ in both worlds
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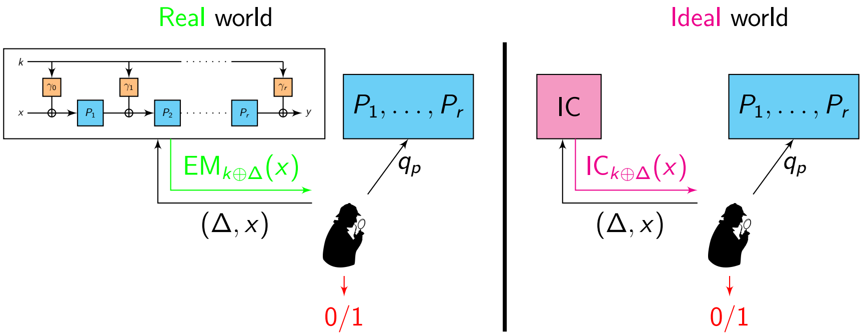

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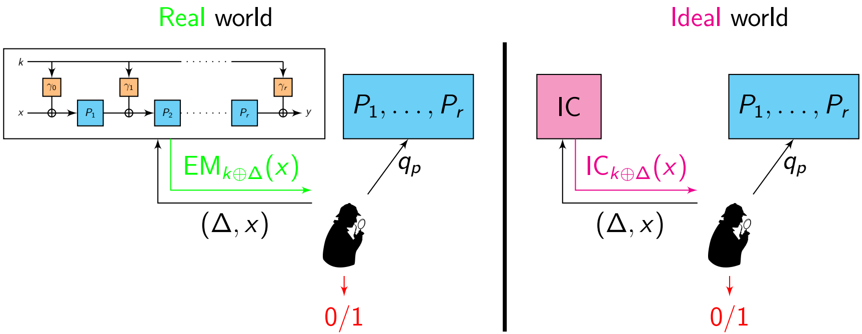

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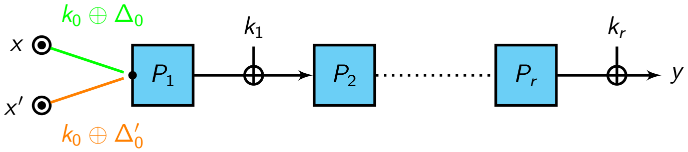
First Observation: Independent Round Keys Fails



RK Distinguisher for independent round keys:

- query $((\Delta_0, 0, \ldots, 0), x)$ and $((\Delta'_0, 0, \ldots, 0), x')$ such that

$$x \oplus \Delta_0 = x' \oplus \Delta'_0$$

- check that the outputs are equal
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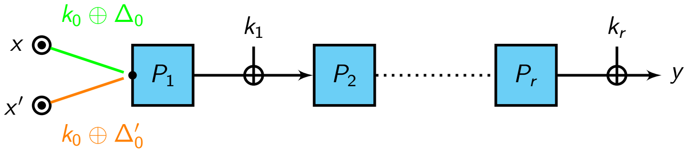

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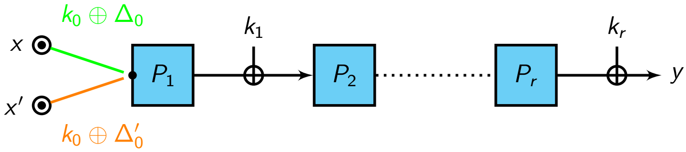

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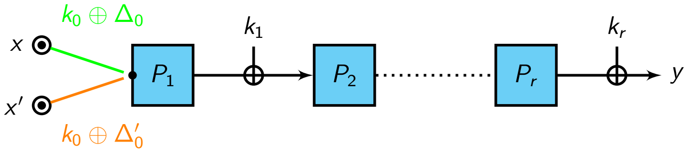

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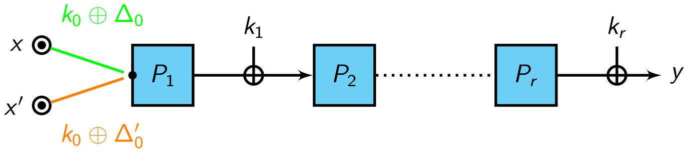

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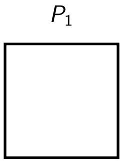

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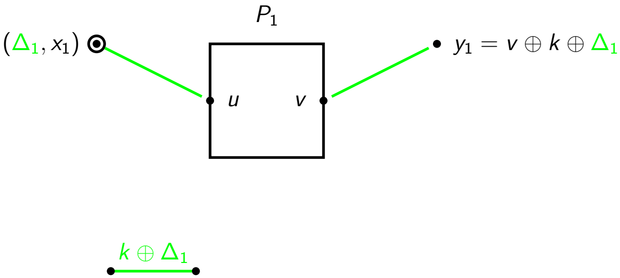

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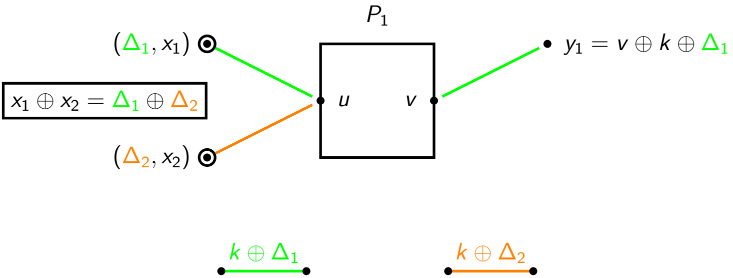

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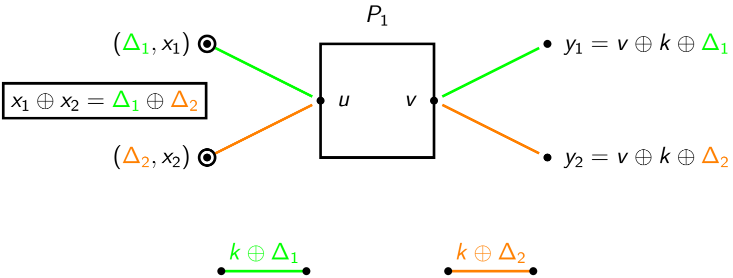

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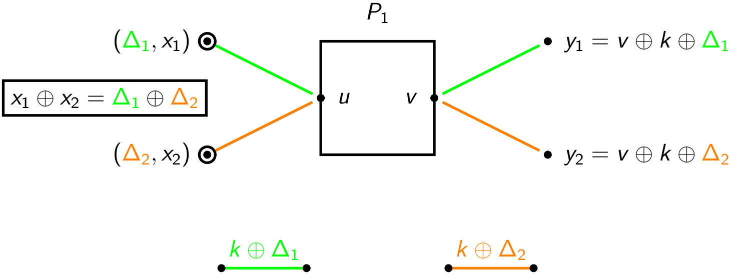

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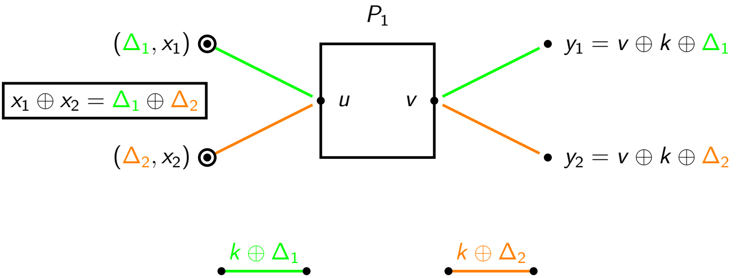

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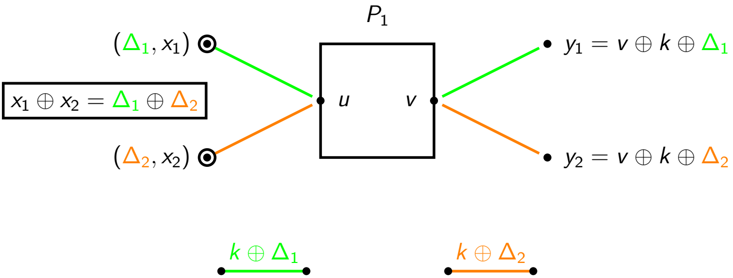

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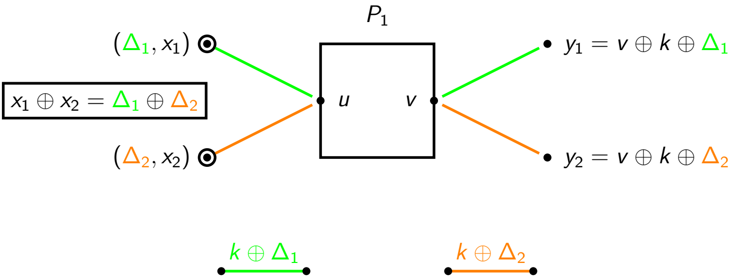

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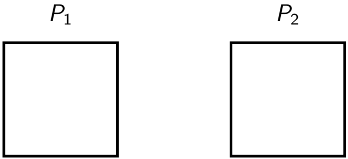

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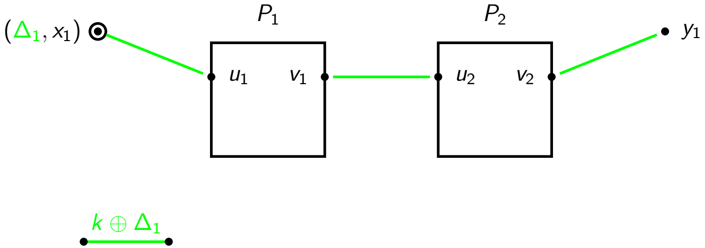

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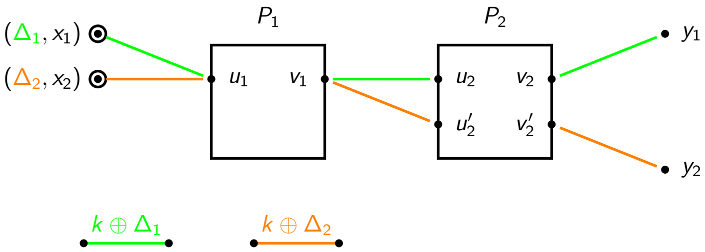

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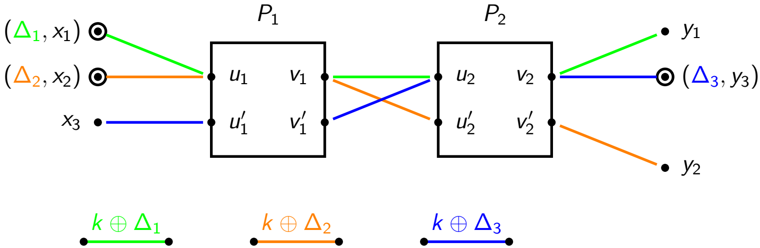

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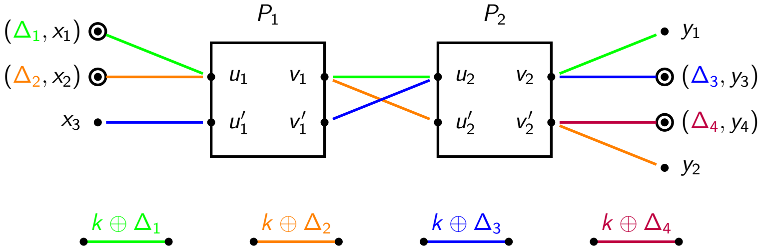

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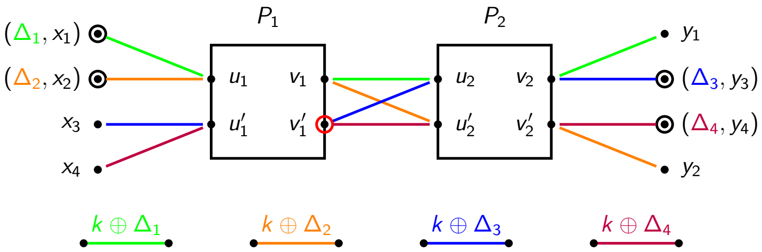

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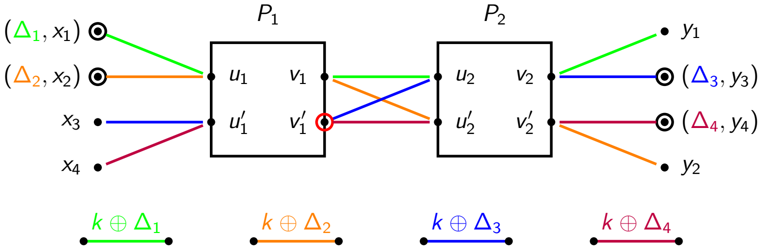

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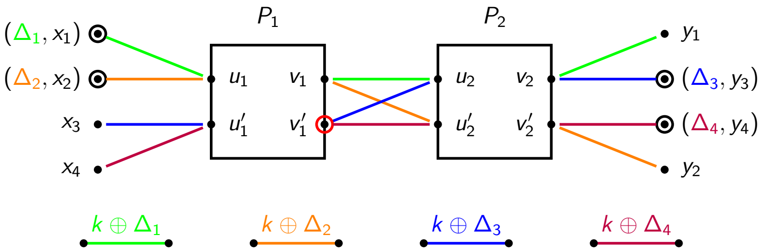

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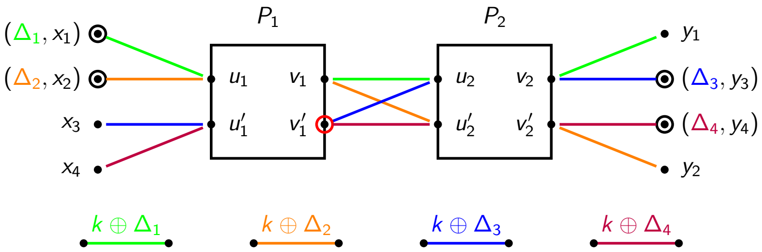

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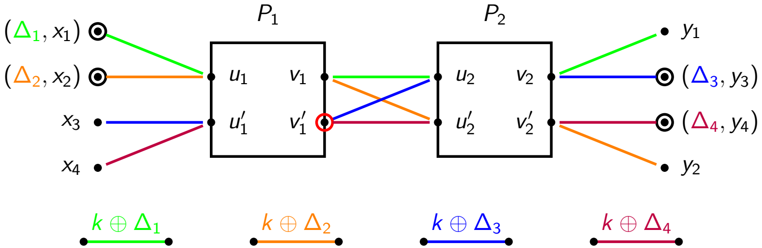

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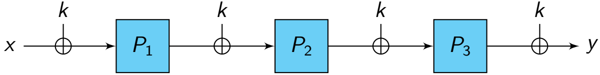
Security for Three Rounds, Trivial Key-Schedule



Theorem (Cogliati-Seurin [CS15])

For the 3-round IEM cipher with the trivial key-schedule:

$$\mathbf{Adv}^{\text{xor-rka}}_{\mathrm{EM}[n,3]}(q_c, q_p) \leq \frac{6q_c q_p}{2^n} + \frac{4q_c^2}{2^n}.$$

Proof sketch:

- $\mathcal{D}$ can create forward collisions at P_1 or backward collisions at P_3
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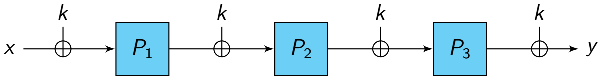

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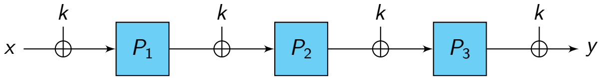

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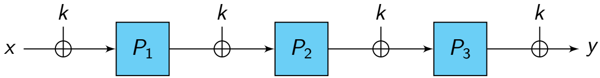

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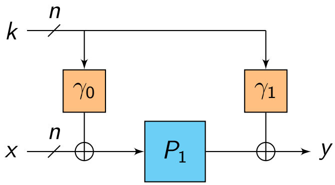
Security for One Round and a Nonlinear Key-Schedule



Theorem (Cogliati-Seurin [CS15])

For the 1-round EM cipher with key-schedule $\gamma = (\gamma_0, \gamma_1)$*:*

$$\mathbf{Adv}^{\text{xor-rka}}_{\text{EM}[n,1,\gamma]}(q_c, q_p) \leq \frac{2q_c q_p}{2^n} + \frac{(\delta(\gamma_0) + \delta(\gamma_1))q_c^2}{2 \cdot 2^n},$$

where $\delta(f) = \max_{a,b \in \{0,1\}^n, a \neq 0} |\{x \in \{0,1\}^n : f(x \oplus a) \oplus f(x) = b\}|$.
($\delta(f) = 2$ *for an APN permutation.)*

Outline

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- informal goal: find tuples of key/pt/ct (k_i, x_i, y_i) with a property which is hard to satisfy for an ideal cipher
- no formal definition for a single, completely instantiated block cipher E
- simply because, e.g., $E_0(0)$ has a specific, non-random value...
- OK this does not count
- but what counts as a chosen-key attack exactly?
- rigorous definition possible for a family of block ciphers based on some underlying ideal primitive
- e.g., IEM cipher based on a tuple of random permutations!

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- informal goal: find tuples of key/pt/ct (k_i, x_i, y_i) with a property which is hard to satisfy for an ideal cipher
- no formal definition for a single, completely instantiated block cipher E
- simply because, e.g., $E_0(0)$ has a specific, non-random value...
- OK this does not count
- but what counts as a chosen-key attack exactly?
- rigorous definition possible for a family of block ciphers based on some underlying ideal primitive
- e.g., IEM cipher based on a tuple of random permutations!

Formalizing Chosen-Key Attacks

Definition (Evasive relation)

An m-ary relation $\mathcal{R}$ is (q, ε)-evasive (w.r.t. an ideal cipher E) if any adversary $\mathcal{A}$ making at most q queries to E finds triples $(k_1, x_1, y_1), \ldots,$ (k_m, x_m, y_m) (with $E_{k_i}(x_i) = y_i$) satisfying $\mathcal{R}$ with probability at most ε.

Example

- consider E in Davies-Meyer mode $f(k, x) := E_k(x) \oplus x$
- finding a preimage of 0 for f is a unary $(q, \mathcal{O}(\frac{q}{2^n}))$-evasive relation for E [BRS02]
- finding a collision for f is a binary $\left(q, \mathcal{O}(\frac{q^2}{2^n})\right)$-evasive relation for E [BRS02]
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Formalizing Chosen-Key Attacks

Definition (Correlation Intractability)

A block cipher construction $\mathcal{C}^F$ based on some underlying primitive F is said to be (q, ε)-correlation intractable w.r.t. an m-ary relation $\mathcal{R}$ if any adversary $\mathcal{A}$ making at most q queries to F finds triples $(k_1, x_1, y_1), \ldots,$ (k_m, x_m, y_m) (with $\mathcal{C}^F_{k_i}(x_i) = y_i$) satisfying $\mathcal{R}$ with probability at most ε.

Definition (Resistance to Chosen-Key Attacks)

Informally, a block cipher construction $\mathcal{C}^F$ is said resistant to chosen-key attacks if for any (q, ε)-evasive relation $\mathcal{R}$, $\mathcal{C}^F$ is (q', ε')-correlation intractable w.r.t. $\mathcal{R}$ with $q' \simeq q$ and $\varepsilon' \simeq \varepsilon$.

Questions:

- How do we prove prove resistance to chosen-key attacks?
- How many rounds for the IEM cipher to be resistant to CKAs?

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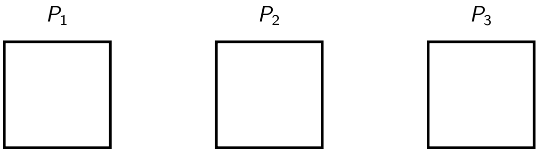
A Chosen-Key Attack for Three Rounds [LS13]



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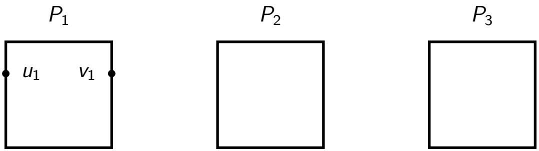

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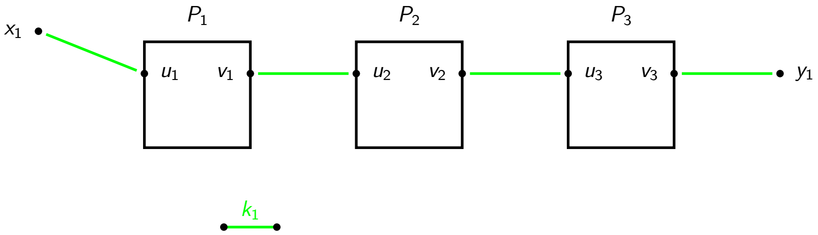

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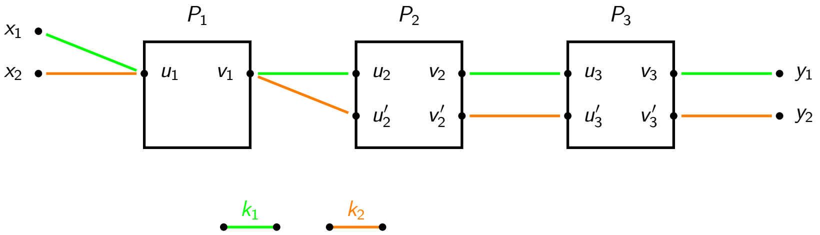

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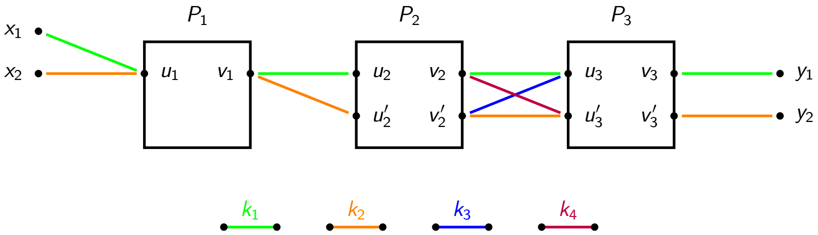

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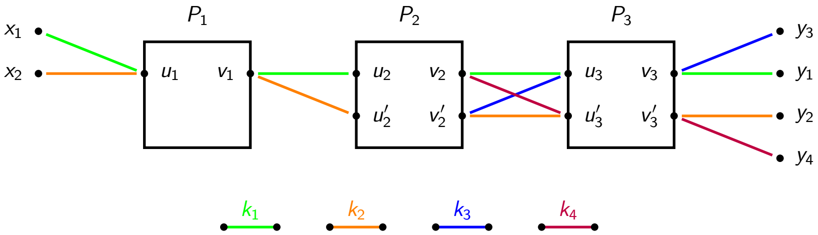

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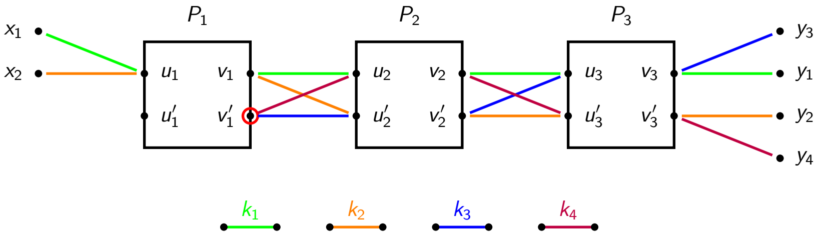

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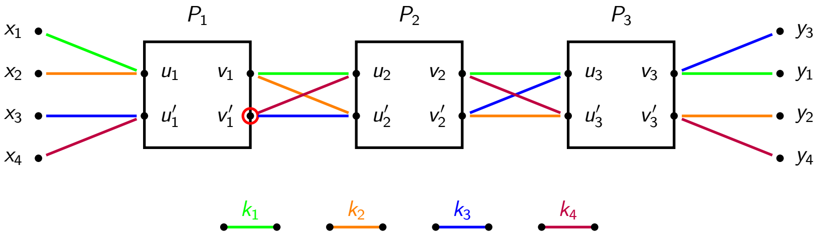

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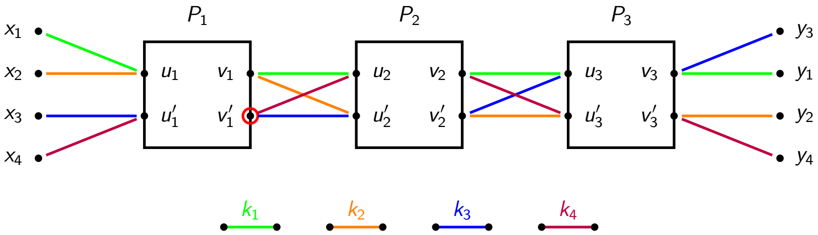

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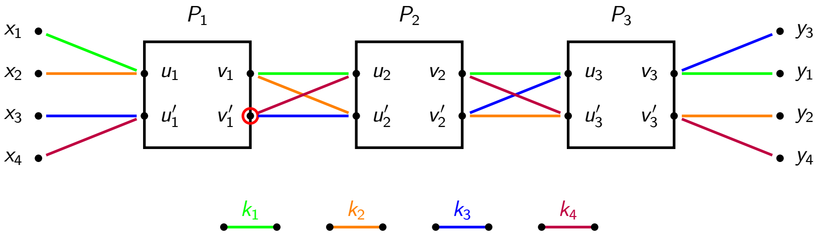

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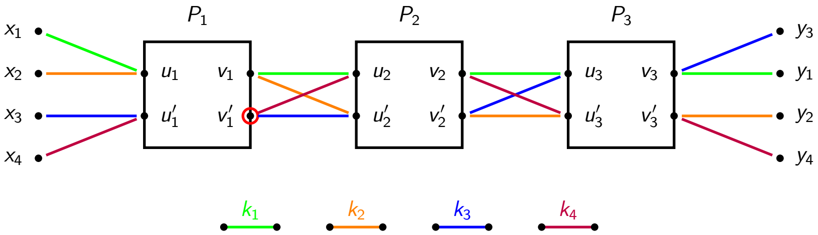

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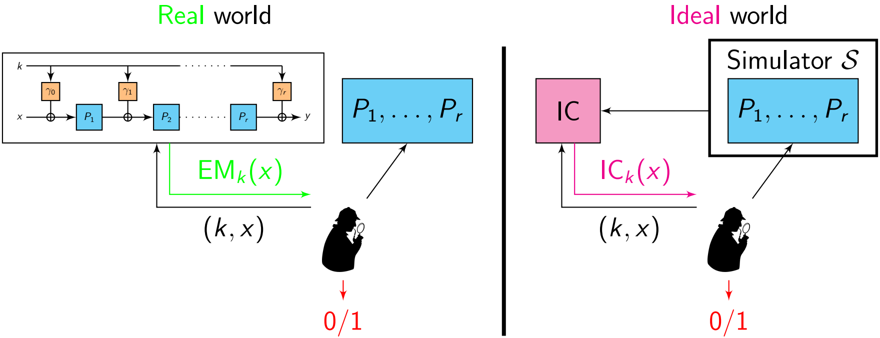
Proving CKA Resistance: Indifferentiability



- real world: IEM cipher + random permutations $P_1, \ldots, P_r$
- ideal world: ideal cipher IC + simulator $\mathcal{S}$
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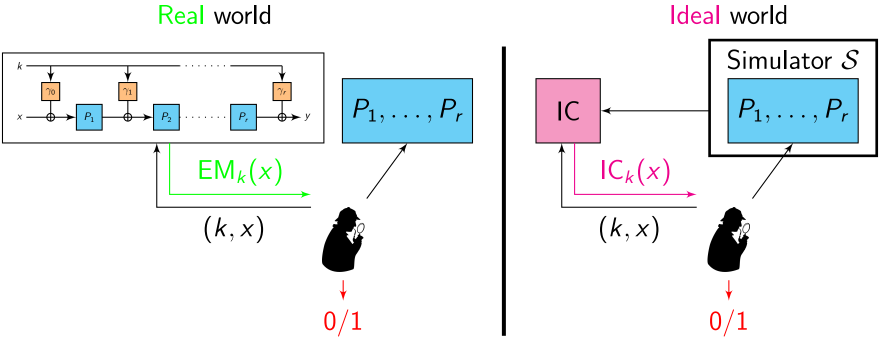

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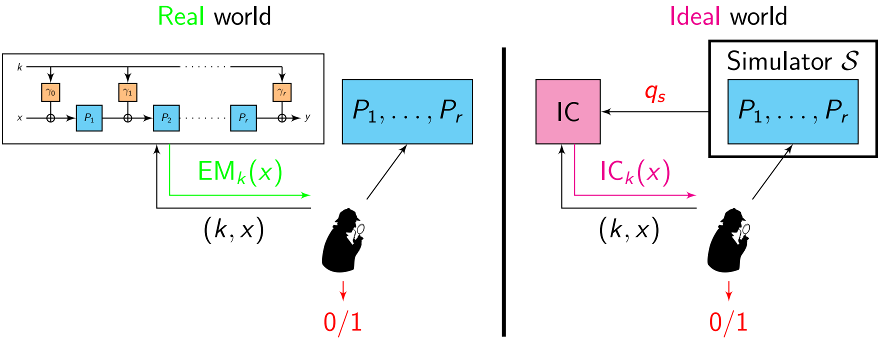
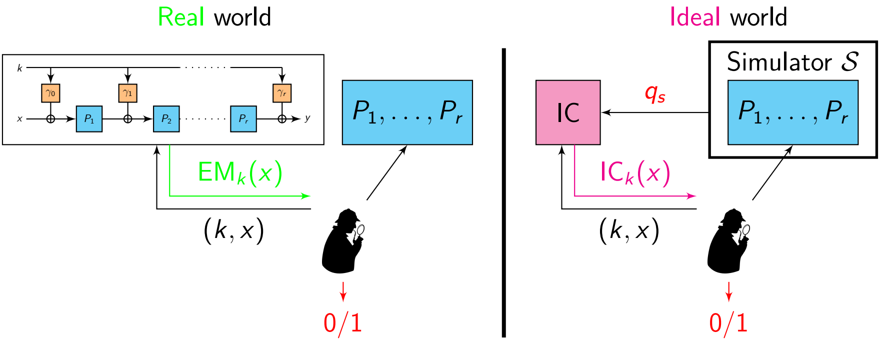


Definition (Indifferentiability [MRH04])

A block cipher construction is said (q_d, q_s, ε)-indifferentiable from an ideal cipher if there exists a simulator $\mathcal{S}$ such that for any distinguisher $\mathcal{D}$ making at most q_d queries in total, $\mathcal{S}$ makes at most q_s ideal cipher queries and $\mathcal{D}$ distinguishes the two worlds with adv. at most ε

Two Flavors of Indifferentiability



- full indifferentiability: $\mathcal{D}$ can queries its oracle as it wishes
- sequential indifferentiability: two query phases
  1. $\mathcal{D}$ first queries only P_i's/$\mathcal{S}$
  2. and then only EM/IC
- full indiff. $\Rightarrow$ sequential indiff.

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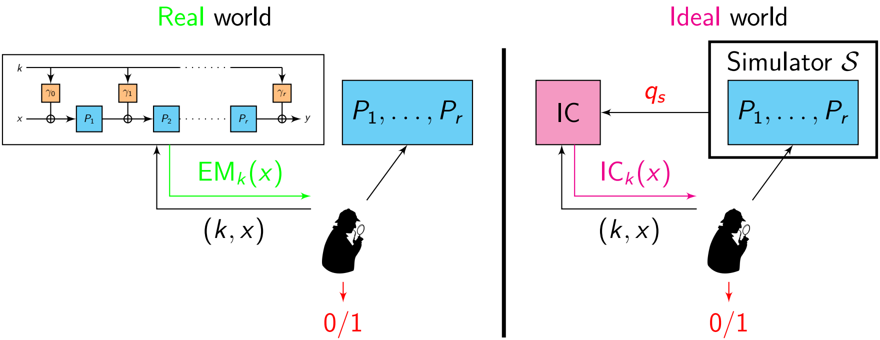

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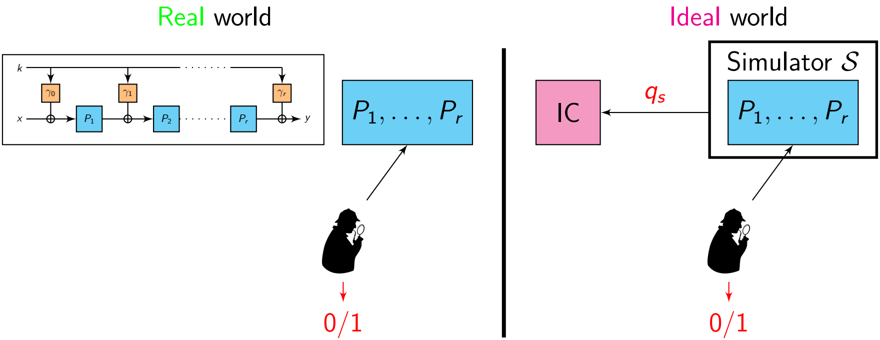

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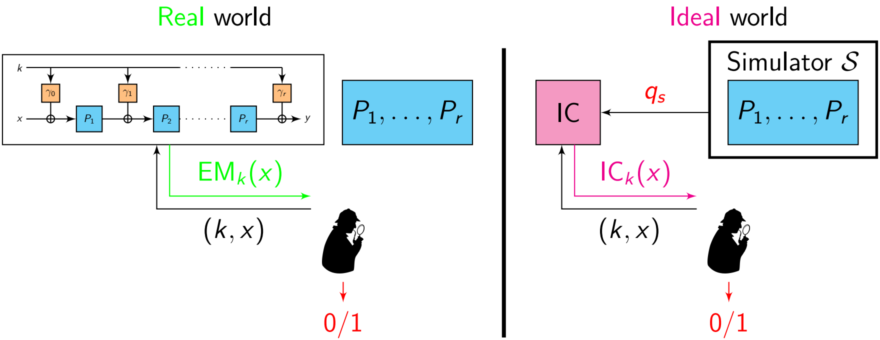

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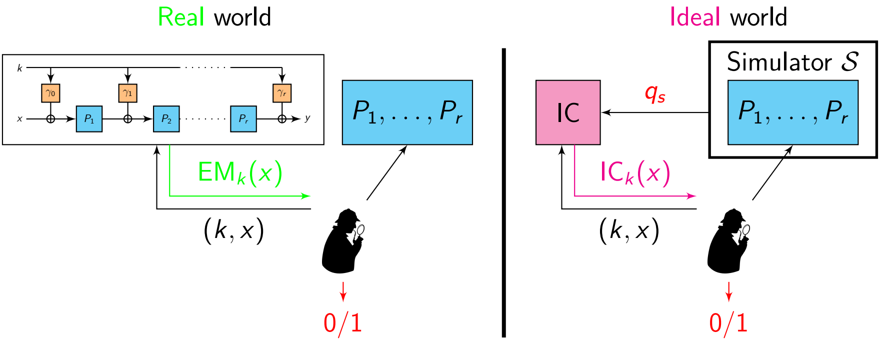

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Theorem (Composition for full indiff. [MRH04])

Informally, if a block cipher construction $\mathcal{C}^F$ is full-indifferentiable from an ideal cipher, then any cryptosystem proven secure with an ideal cipher remains provably secure when used with $\mathcal{C}^F$ (for cryptosystems whose security is defined by a single-stage game [RSS11]).

Theorem ([MPS12, CS15])

If a block cipher construction $\mathcal{C}^F$ is (q_d, q_s, ε)-seq-indiff. from an ideal cipher, and if a relation $\mathcal{R}$ is $(q_s, \varepsilon_{\mathrm{ic}})$-evasive for an ideal cipher, then $\mathcal{C}^F$ is $(q_d, \varepsilon_{\mathrm{ic}} + \varepsilon)$-correlation intractable w.r.t. $\mathcal{R}$.

	IC		$\mathcal{C}^F$
queries	q_s	$\xrightarrow{(q_d, q_s, \varepsilon)\text{-seq-indiff.}}$	q_d
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Composition Theorems

Theorem (Composition for full indiff. [MRH04])

Informally, if a block cipher construction $\mathcal{C}^F$ is full-indifferentiable from an ideal cipher, then any cryptosystem proven secure with an ideal cipher remains provably secure when used with $\mathcal{C}^F$ (for cryptosystems whose security is defined by a single-stage game [RSS11]).

Theorem ([MPS12, CS15])

If a block cipher construction $\mathcal{C}^F$ is (q_d, q_s, ε)-seq-indiff. from an ideal cipher, and if a relation $\mathcal{R}$ is $(q_s, \varepsilon_{\mathrm{ic}})$-evasive for an ideal cipher, then $\mathcal{C}^F$ is $(q_d, \varepsilon_{\mathrm{ic}} + \varepsilon)$-correlation intractable w.r.t. $\mathcal{R}$.

	IC		$\mathcal{C}^F$
queries	q_s	$\xrightarrow{(q_d, q_s, \varepsilon)\text{-seq-indiff.}}$	q_d
success proba.	$\varepsilon_{\mathrm{ic}}$		$\varepsilon_{\mathrm{ic}} + \varepsilon$

Indifferentiability Results for the IEM Cipher

Theorem (Andreeva *et al.* [ABD+13])

The 5-round IEM cipher with a key-schedule modeled as a random oracle is fully indifferentiable from an ideal cipher.

NB: strong assumption on the key-schedule (often invertible in real BCs)

Theorem (Lampe-Seurin [LS13])

The 12-round IEM cipher with the trivial key-schedule is fully indifferentiable from an ideal cipher.

Theorem (Cogliati-Seurin [CS15])

The 4-round IEM cipher with the trivial key-schedule is sequentially indifferentiable from an ideal cipher with $q_s = \mathcal{O}(q_d^2)$ and $\varepsilon = \mathcal{O}(q_d^4/2^n)$

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CKA Resistance for the 4-Round IEM Cipher

By the composition theorem "seq-indiff. $\Rightarrow$ correlation-intractability":

Theorem

Let $\mathcal{R}$ be a $(q^2, \varepsilon_{\mathrm{ic}})$-evasive relation w.r.t. an ideal cipher. Then the 4-round IEM with the trivial key-schedule is $\left(q, \varepsilon_{\mathrm{ic}} + \mathcal{O}(\frac{q^4}{2^n})\right)$ correlation intractable w.r.t. $\mathcal{R}$.

Example

Consider f = 4-round IEM cipher in Davies-Meyer mode. Then

- f is $\left(q, \mathcal{O}(\frac{q^4}{2^n})\right)$-preimage resistant
- f is $\left(q, \mathcal{O}(\frac{q^4}{2^n})\right)$-collision resistant

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Conclusion

Morality:

- idealized models can be fruitful
- practical meaning of the results is debatable:
  - the high-level structure of SPNs is sound (and may even yield something close to an ideal cipher)
  - says little about concrete block ciphers (inner permutations of, say, AES are too simple)

Open problems:

- pseudorandomness for non-independent round keys, $r \geq 3$
- full indifferentiability:
  - best known attack is only on 3 rounds (for trivial KS)
  - minimal number of rounds for full indifferentiability ? ($4 \leq r \leq 12$)
  - $\Rightarrow$ the 4-round IEM might already be fully indifferentiable from an IC

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Summary of Known Results

Security notion	# of rounds	Key schedule	Security bound	Simul. (q_S/t_S)	Ref.
Single-key	$r \geq 1$	independent	$2^{\frac{rn}{r+1}}$	—	[CS14]
	1	trivial	$2^{\frac{n}{2}}$	—	[EM97, DKS12]
	2	trivial	$2^{\frac{2n}{3}}$	—	[CLL+14]
XOR RKA	3	trivial	$2^{\frac{n}{2}}$	—	[CS15, FP15]
	1	nonlinear	$2^{\frac{n}{2}}$	—	[CS15]
CKA (Seq-ind.)	4	trivial	$2^{\frac{n}{4}}$	q^2 / q^2	[CS15]
Full indiff.	5	rand. oracle	$2^{\frac{n}{10}}$	q^2 / q^3	[ABD+13]
	12	trivial	$2^{\frac{n}{12}}$	q^4 / q^6	[LS13]

The End...

Thanks for your attention!

Comments or questions?

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