Minimizing the Two-Round Even-Mansour Cipher

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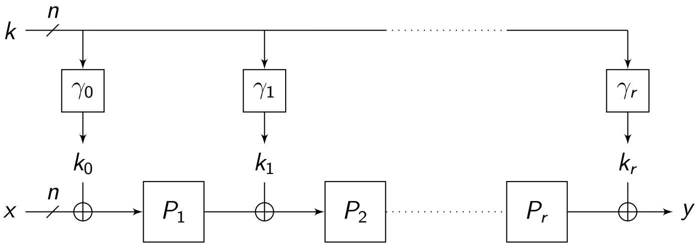
Outline

1. Context: Security Proofs for Key-Alternating Ciphers
2. Overview of our Results
3. Sketch of the Security Proof

Key-alternating ciphers



An r-round key-alternating cipher

- $k \in \{0,1\}^n$ is the (master) key, x the plaintext, y the ciphertext
- The P_i's are public permutations on $\{0,1\}^n$
- The γ_i's are key derivation functions mapping k to n-bit "round keys"
- prominent example: AES-128

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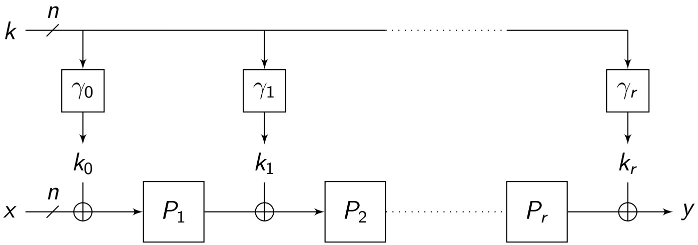

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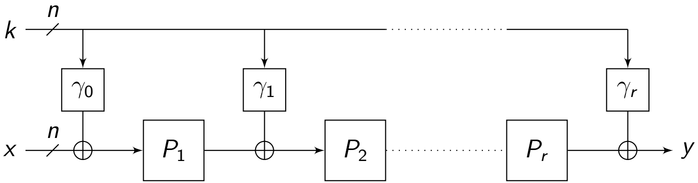

Question

How can we "prove" security? (for this talk, security = pseudorandomness)

- against a general adversary: too hard! (unconditional complexity lower bound)
- against specific attacks (differential, linear...): use specific design of $P_1, \ldots, P_r$, count active S-boxes, etc.
- against generic attacks: Random Permutation Model for $P_1, \ldots, P_r$

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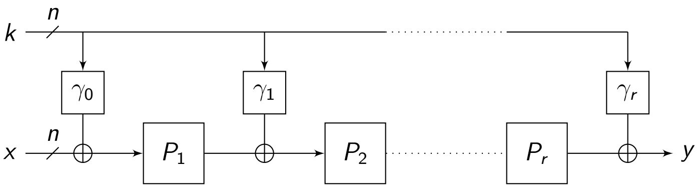

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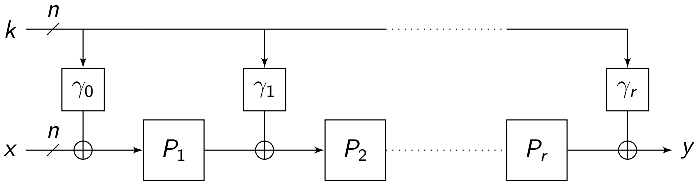

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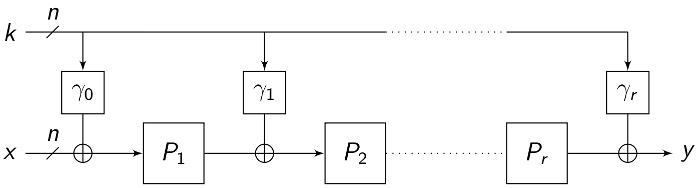

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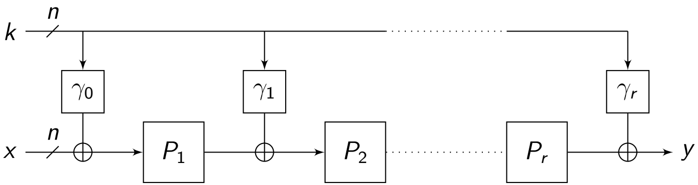
Analyzing KA ciphers in the Random Permutation Model



- the P_i's are viewed as public random permutation oracles to which the adversary can only make black-box queries (both to P_i and P_i^{-1}).
- trades complexity for randomness and allows for a completely information-theoretic proof ($\simeq$ Random Oracle Model)
- complexity measure of the adversary:
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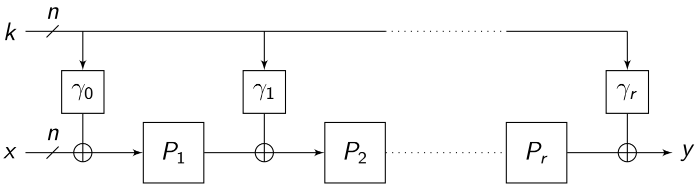

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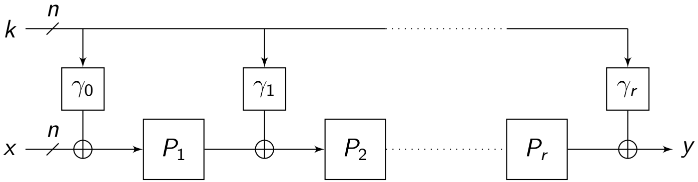

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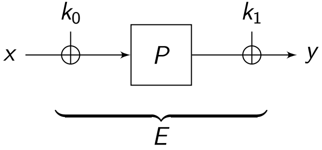

Similar result when $k_0 = k_1$ [DKS12]

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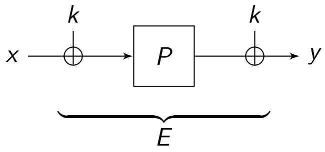

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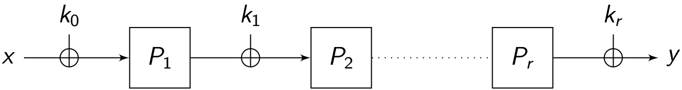

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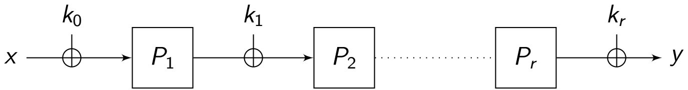
1. independent round keys $(k_0, k_1, \ldots, k_r)$,
2. independent inner permutations $P_1, \ldots, P_r$,

KA ciphers are secure against generic attacks as long as

$$q_e \text{ and } q_p \ll \mathcal{O}(2^{\frac{rn}{r+1}}).$$

This result is tight (in terms of query complexity).

State of the art



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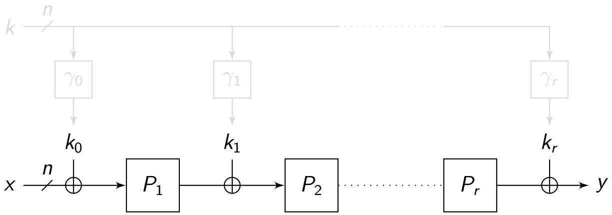
Our problem

Main question

Is it possible to prove a similar $\mathcal{O}(2^{\frac{rn}{r+1}})$ bound when:

- the round keys $(k_0, \ldots, k_r)$ are derived from an n-bit master key
- and/or when the same permutation P is used at each round

as is the case in many concrete designs (AES-128, etc.)?



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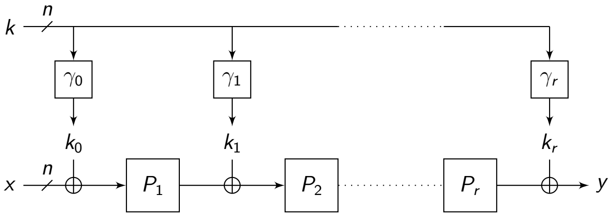

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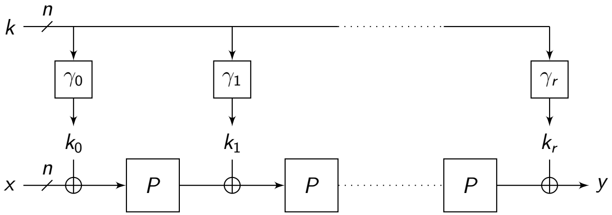

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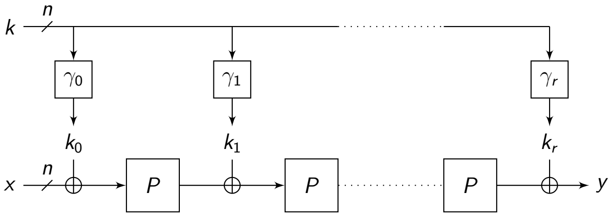

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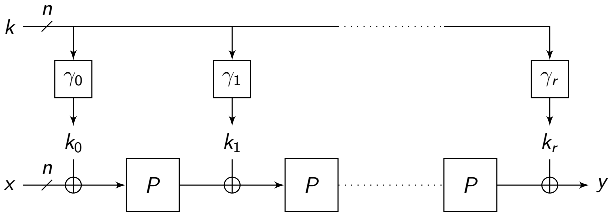

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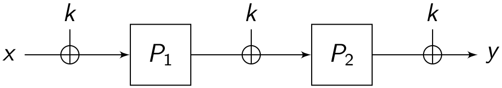
Our results (1/2): two independent permutations

First, we deal with the (simpler) case where the two inner permutations are independent. Then the trivial key-schedule is sufficient.

Theorem

The 2-round EM cipher with independent random permutations and identical round keys is secure up to $\widetilde{\mathcal{O}}(2^{\frac{2n}{3}})$ queries of the adversary.



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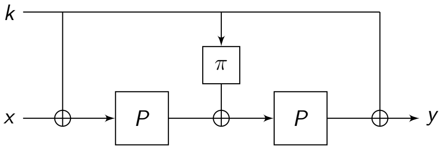

π can be any fixed ($\mathbb{F}_2$-linear) orthomorphism (i.e., π is a permutation and $k \mapsto k \oplus \pi(k)$ is a permutation), for instance

$$\pi : (k_L, k_R) \mapsto (k_R, k_L \oplus k_R) \quad \text{(Feistel)}$$
$$\pi : k \mapsto c \odot k, \quad \text{for } c \neq 0, 1 \quad \text{(field mult.)}$$

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Theorem (more general)

The 2-round EM cipher below is secure up to $\widetilde{\mathcal{O}}(2^{\frac{2n}{3}})$ queries when

(i) $\gamma_0, \gamma_1, \gamma_2$ *are* $\mathbb{F}_2$*-linear permutations;*

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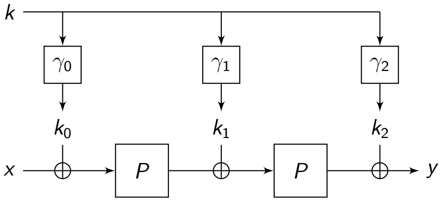

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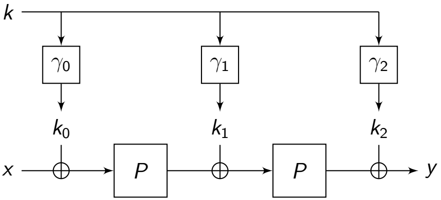

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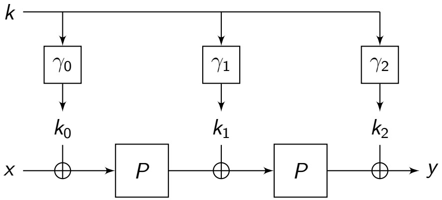

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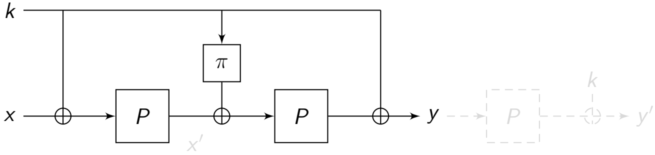

This construction is "minimal" to achieve $\mathcal{O}(2^{\frac{2n}{3}})$ security.
Removing any component causes security to drop back to $\mathcal{O}(2^{\frac{n}{2}})$:

- removing one of the P's: 1-round Even-Mansour, $\mathcal{O}(2^{\frac{n}{2}})$-secure
- removing π: slide attack with $\mathcal{O}(2^{\frac{n}{2}})$ complexity:
  - find $(x, y), (x', y')$ such that $x' = P(x \oplus k)$ (slid pair)
  - can be detected by checking that $x \oplus P(y) = y' \oplus P^{-1}(x')$
  - works for any number of rounds for id. round keys and id. permutations

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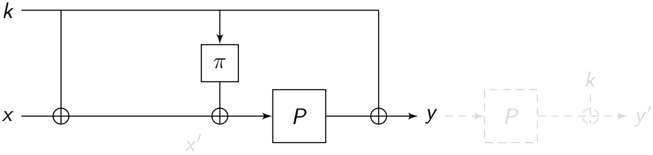

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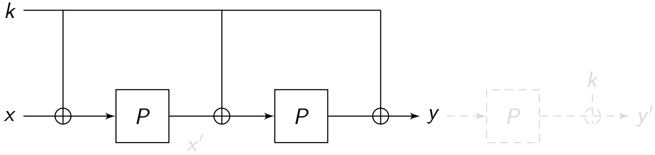

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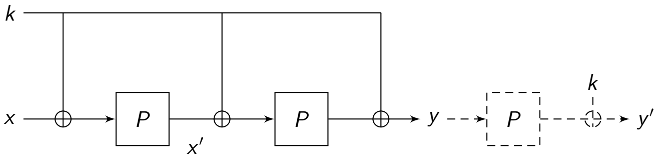

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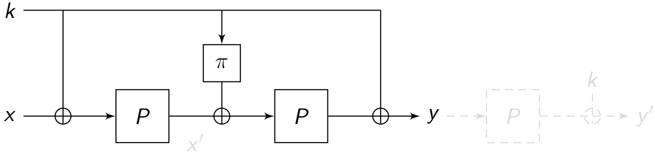

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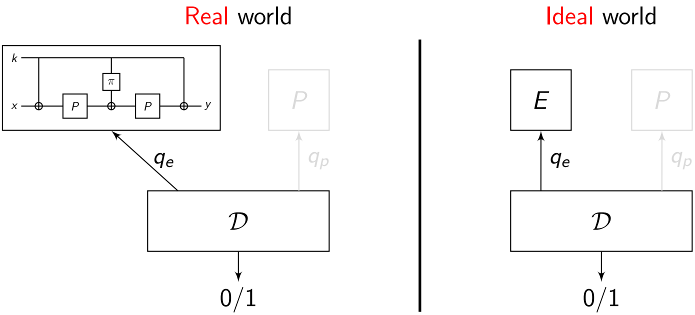
Formalizing indistinguishability (in the RP Model)



- real world: cipher with a random key $k \leftarrow_{\$} \{0,1\}^n$
- ideal world: E is a random permutation independent from P
- Random Permutation Model: $\mathcal{D}$ has oracle access to P in both worlds
- for this talk, $q_e = q_p = q$

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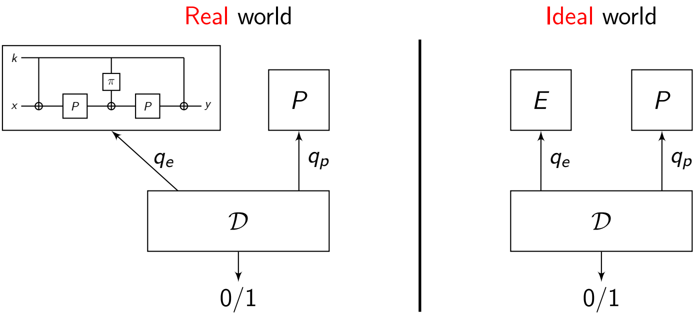

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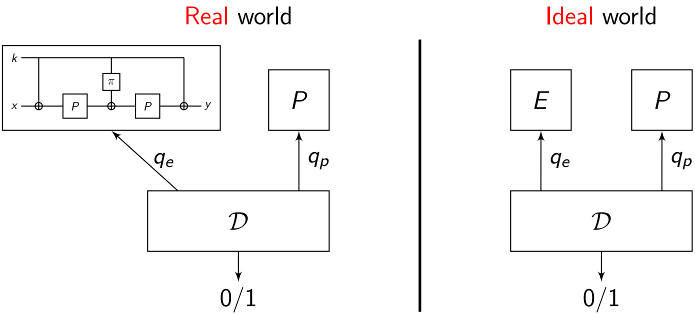

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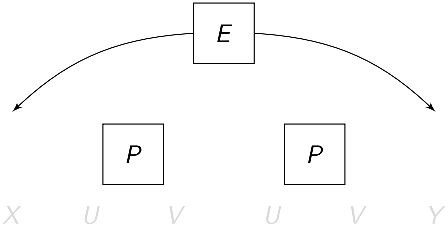
Query transcript



The distinguisher can query:

- oracle E forward: $E(x) = y$, and backward: $E^{-1}(y) = x$
- oracle P forward: $P(u) = v$, and backward: $P^{-1}(v) = u$

This results in a query transcript $\tau = (\mathcal{Q}_E, \mathcal{Q}_P)$:

$$\mathcal{Q}_E = \{(x_1, y_1), \ldots, (x_q, y_q)\}$$
$$\mathcal{Q}_P = \{(u_1, v_1), \ldots, (u_q, v_q)\}.$$

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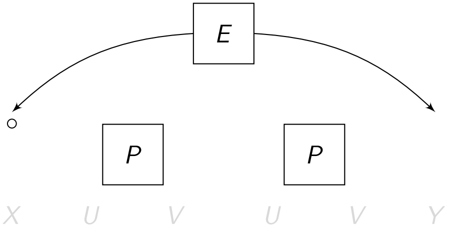

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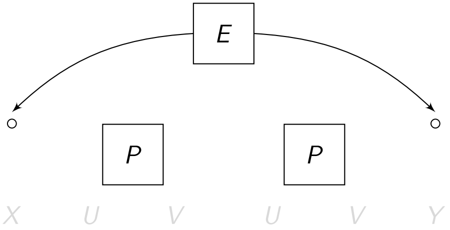

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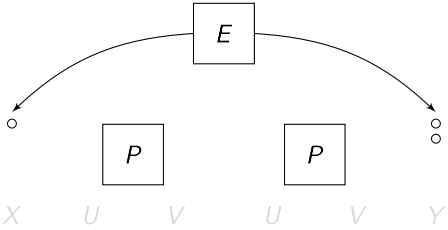

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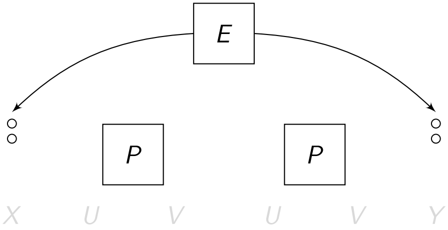

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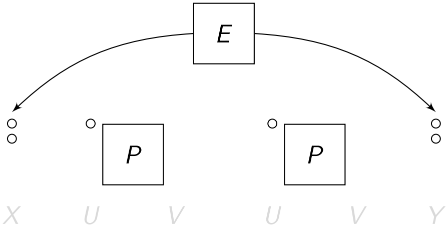

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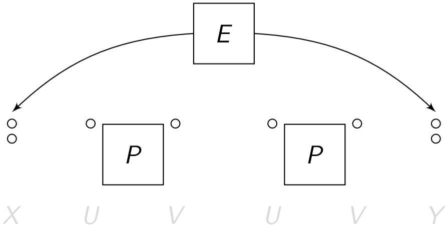

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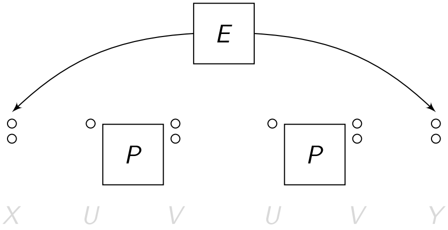

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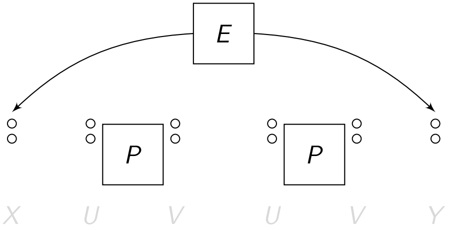

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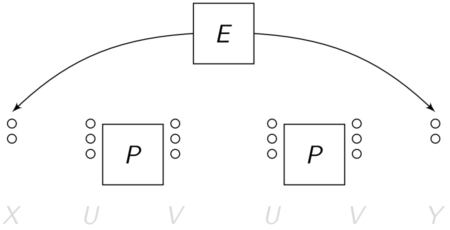

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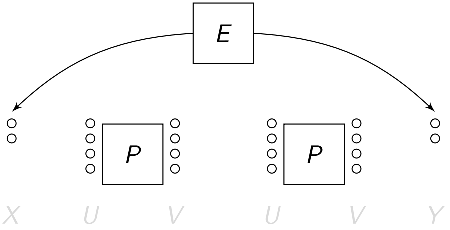

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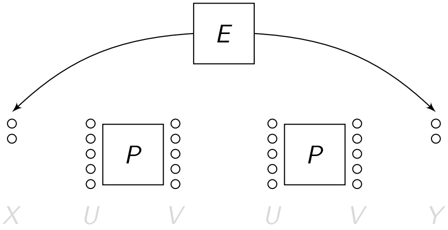

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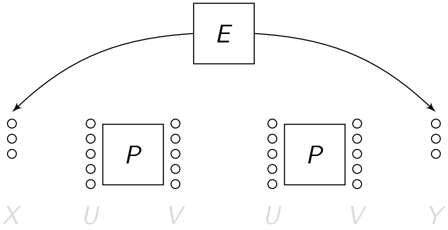

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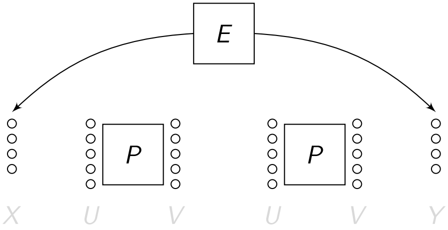

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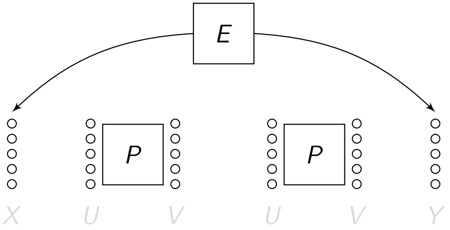

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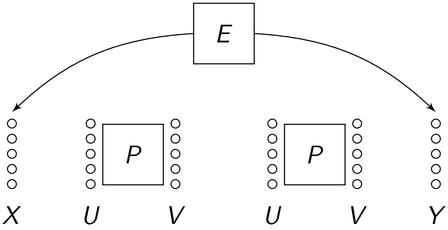

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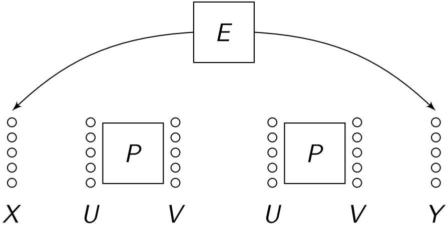
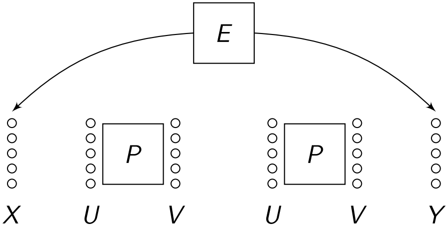
H-coefficient framework



$$\mathbf{Adv}(\mathcal{D}) \leq \| T_{\mathrm{real}} - T_{\mathrm{ideal}} \| \quad \text{(statistical distance)}$$

$$T_{\mathrm{real/ideal}} = \text{distribution of transcript } (\mathcal{Q}_E, \mathcal{Q}_P)$$
$$\text{in the real/ideal world}$$

H-coefficient framework

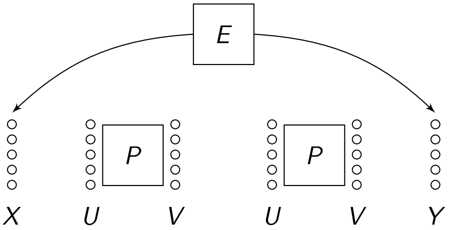


Lemma

Partition the set of transcripts into "good" ones $\mathcal{T}_{\text{good}}$ and "bad" ones $\mathcal{T}_{\text{bad}}$. Then

$$\left.\begin{array}{l} \forall \tau \in \mathcal{T}_{\text{good}}, \frac{\Pr[T_{\text{real}}=\tau]}{\Pr[T_{\text{ideal}}=\tau]} \geq 1-\varepsilon_1 \\ \\ \Pr[T_{\text{ideal}} \in \mathcal{T}_{\text{bad}}] \leq \varepsilon_2 \end{array}\right\} \Rightarrow \mathbf{Adv}(\mathcal{D}) \leq \varepsilon_1 + \varepsilon_2$$

Bad keys and bad transcripts (simplified)



A key k' is bad if $\mathcal{D}$ can check its "compatibility" with the transcript:

1. $\exists(x, y) \in \mathcal{Q}_E$, $u \in U$, $v \in V$: $k' = x \oplus u = y \oplus v$
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A transcript $(\mathcal{Q}_E, \mathcal{Q}_P)$ is bad if it has too many bad keys.

We must show that with high probability,

$$\# \text{ bad keys} \ll 2^n.$$

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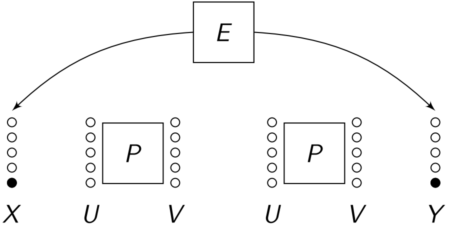

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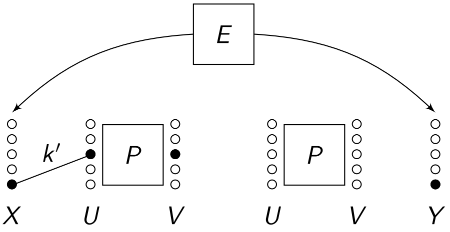

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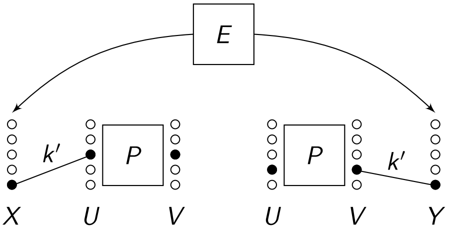

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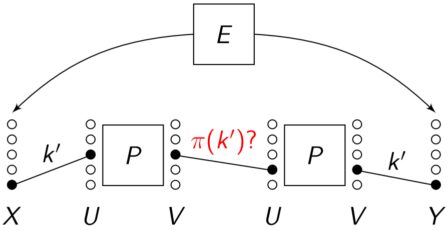

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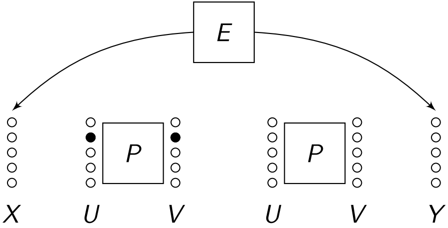

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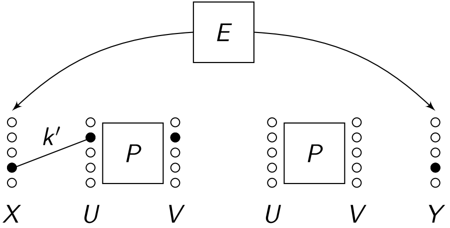

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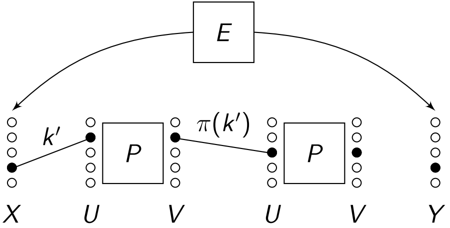

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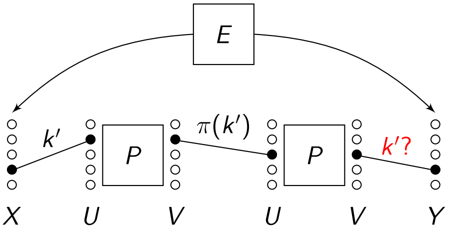

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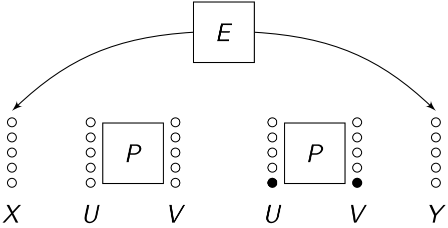

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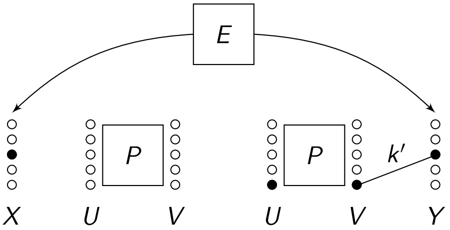

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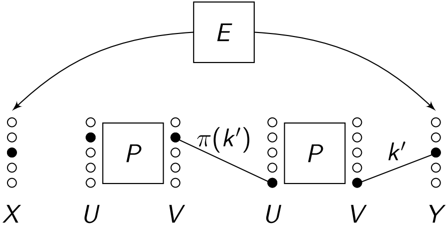
A transcript $(\mathcal{Q}_E, \mathcal{Q}_P)$ is bad if it has too many bad keys.

We must show that with high probability,

$$\#\ \text{bad keys} \ll 2^n.$$

Bad keys and bad transcripts (simplified)



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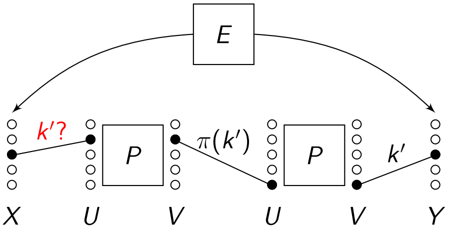

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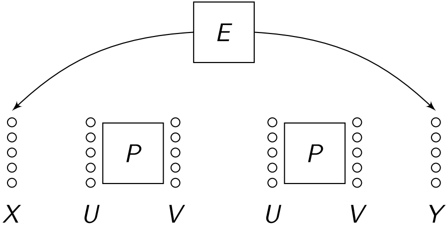

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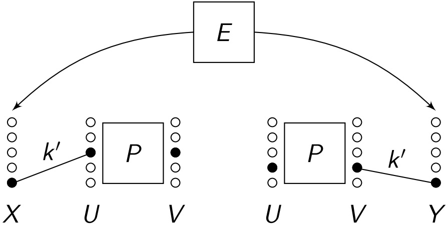

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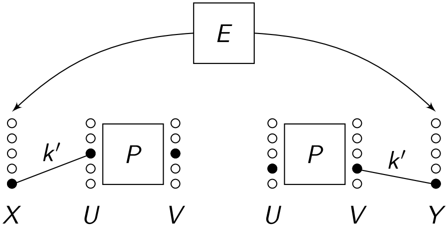

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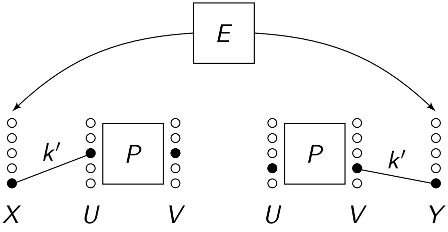

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The sum-capture problem

For $A = \{a_1, \ldots, a_q\} \subseteq \{0,1\}^n$, let

$$\mu(A) = \max_{\substack{U,V \subseteq \{0,1\}^n \\ |U|=|V|=q}} |\{(a,u,v) \in A \times U \times V : a = u \oplus v\}|$$

If A is "structured", e.g. a vector space, then $\mu(A) = q^2$

Sum-capture problem: find upper bounds on $\mu(A)$ for a random set A

Theorem ([Bab89, Ste13])

For $q \leq 2^{\frac{2n}{3}}$, then with overwhelming probability for a random set A,

$$\mu(A) \lesssim q^{\frac{3}{2}}.$$

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Permutation counting problem (simplified)

Let $X = \{x_1, \ldots, x_q\}$ and $Y = \{y_1, \ldots, y_q\}$ with $X \cap Y$ "small". Compare

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Assume $|X \cap Y| \leq q/2^{n/3}$. Then $p_{\text{real}} \geq (1 - \varepsilon_1)\, p_{\text{ideal}}$ with $\varepsilon_1 = \mathcal{O}\left(\frac{q^3}{2^{2n}}\right)$.

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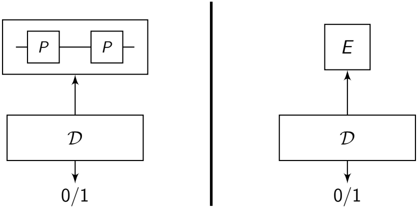
Random *square* permutation vs. random permutation



Random Square Permutation Problem

How many queries needs $\mathcal{D}$ to distinguish a random square permutation $P \circ P$ from a perfectly random permutation E?

Conjecture: indistinguishable up to $\sim 2^n$ queries

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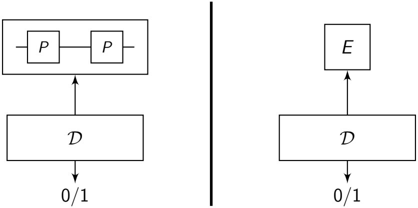

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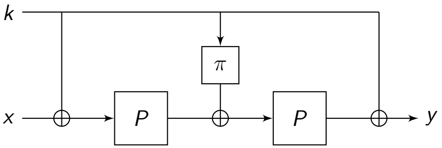

- first "beyond birthday-bound" security result for AES-like ciphers that does not require the "independent round keys" assumption
- open problems:
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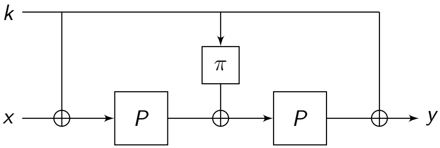

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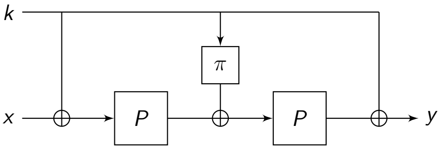

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The end...

Thanks for your attention!

Comments or questions?

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