Counter-in-Tweak: Authenticated Encryption Modes for Tweakable Block Ciphers

Thomas Peyrin[1] Yannick Seurin[2]

[1]NTU, Singapore

[2]ANSSI, France

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Context

- starting point: CAESAR competition for Authenticated Encryption (AE)
- more precisely, candidates Deoxys, Joltik and KIASU (Jean, Nikolic, Peyrin)
- each is based on a tweakable block cipher (Deoxys-BC, Joltik-BC, or KIASU-BC) and two modes of operation:
  - ΘCB for the nonce-respecting setting
  - COPA for the nonce-misuse setting
- problems with COPA:
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  - for fractional messages, relied on XLS which has been broken [Nan14]

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Our Goal

- in replacement of COPA, design an AE mode of operation for tweakable block ciphers which provides:
  1. (full, not online) nonce-misuse resistance up to the birthday bound
  2. beyond-birthday-bound (BBB) security in the nonce-respecting setting
- existing (TBC $\Rightarrow$ AE) modes:
  - ΘCB [KR11] is perfectly secure in the nonce-respecting scenario, but not secure at all in the nonce-misuse scenario
  - COPA [ABL+13] provides only *online* nonce-misuse resistance
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  - PIV [ST13] requires a very long tweak-length (size of the maximal message length)
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TBCs and AE

Generic Composition: the NSIV Method

Authentication: the EPWC Mode

Encryption: the CTRT Mode

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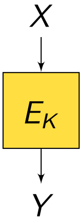

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- T assumed public or even adversarially controlled
- each tweak should give an “independent” permutation
- few “natively tweakable” BCs:
    - Hasty Pudding Cipher [Sch98]
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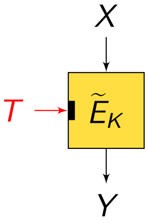

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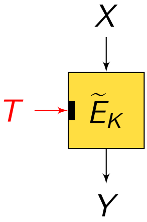

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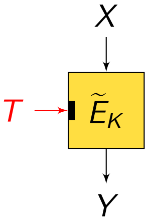

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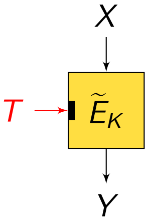

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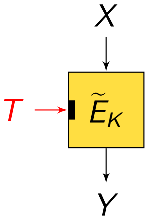

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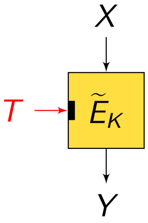

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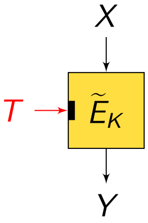

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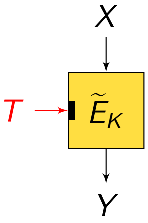

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Goal: Nonce-Based Authenticated Encryption (nAE)

Syntax

A nAE scheme Π is a pair of algorithms (Π.Enc, Π.Dec) where

- algorithm Π.Enc takes
  - (a key K)
  - a nonce N
  - associated data A
  - a message M

  and returns a ciphertext C.
- algorithm Π.Dec takes K and (N, A, C) and returns M or $\perp$.

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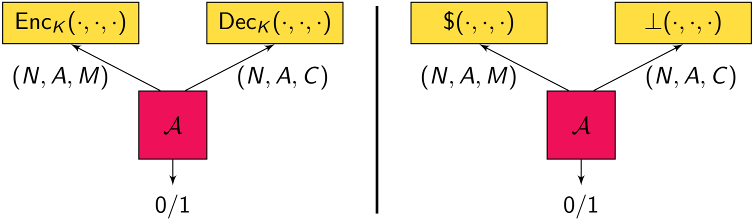

Security (all-in-one definition)

- The scheme Π is secure if adversary $\mathcal{A}$ cannot distinguish $(\mathrm{Enc}_K, \mathrm{Dec}_K)$ and $(\$, \perp)$.
- $\mathcal{A}$ cannot ask a decryption query (N, A, C) if it received C from an encryption query (N, A, M)
- $\mathcal{A}$ is said nonce-respecting if it never repeats a nonce in encryption queries.

Misuse-Resistant AE (MRAE)

Nonce-misuse resistance (informal) [RS06]

A nAE scheme is said nonce-misuse resistant if repeating a nonce in encryption queries:

- does not harm authenticity
- hurts confidentiality only insofar as repetitions of triplets (N, A, M) are detectable

- $\simeq$ deterministic authenticated encryption
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Generic Composition

Starting from two building blocks:

- a MAC (or a PRF) $F_{K_1}(\cdot,\cdot,\cdot)$
- an encryption scheme $\mathrm{Enc}_{K_2}(\cdot,\cdot)$

combine them to obtain a nAE scheme [BN00, NRS14].

Two types of encryption schemes:

- (random) IV-based encryption (ivE): $C = \mathrm{Enc}_{K_2}(IV, M)$, IV randomly chosen by the encryption oracle (ex: CBC)
- nonce-based encryption (nE): $C = \mathrm{Enc}_{K_2}(N, M)$, N chosen by the adversary but non-repeating (ex: nonce-based CTR mode, GCM)

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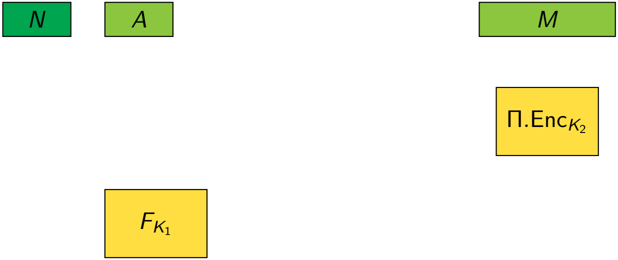

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- what about BBB-security in the nonce-respecting case?

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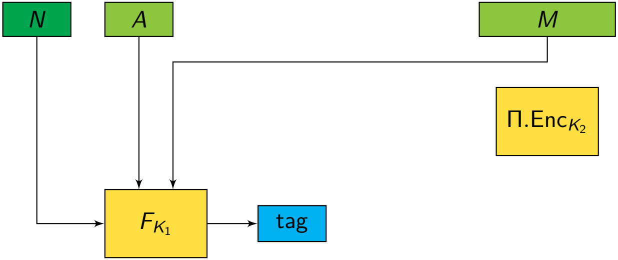

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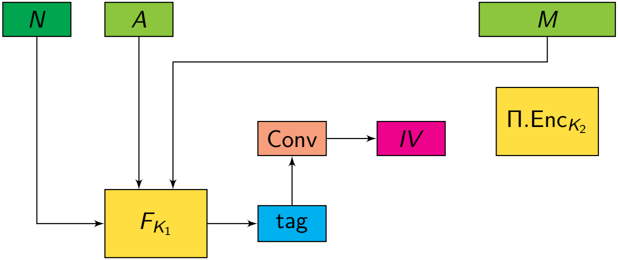

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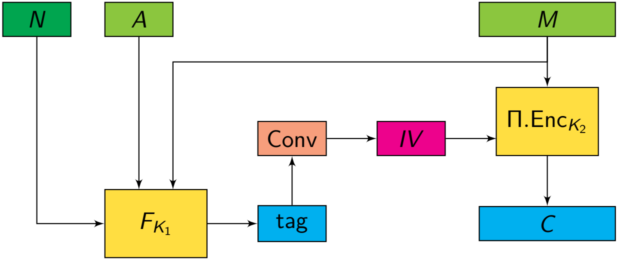

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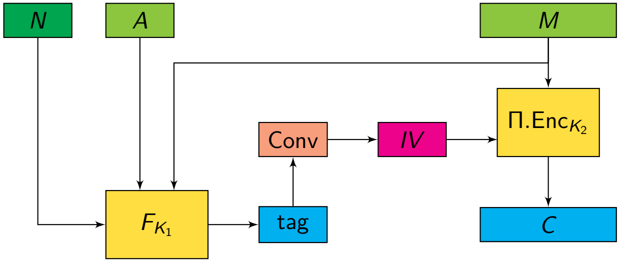

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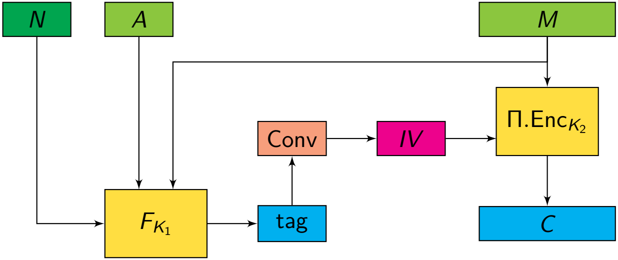

- SIV (*Synthetic IV*) [RS06] combines a PRF $F_{K_1}(N, A, M)$ and an IV-based encryption scheme $\Pi.\mathrm{Enc}_{K_2}(IV, M)$
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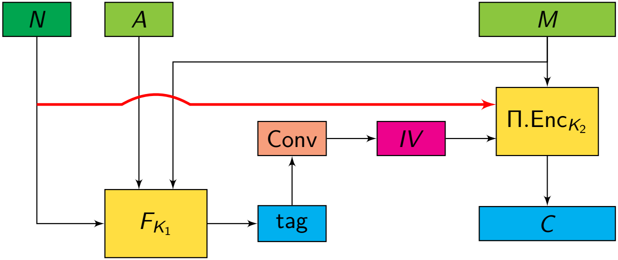

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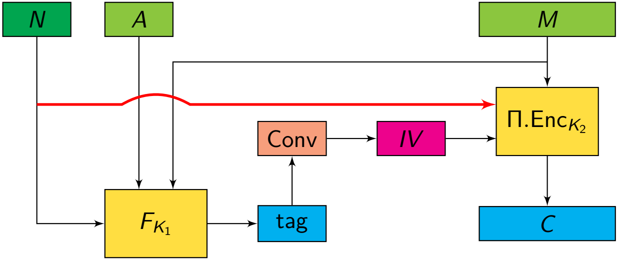
  $\Rightarrow$ Re-use the nonce N in the encryption scheme!

Combined Nonce and IV-based (nivE) Encryption



- the encryption algorithm Π.Enc takes a nonce and a random IV!
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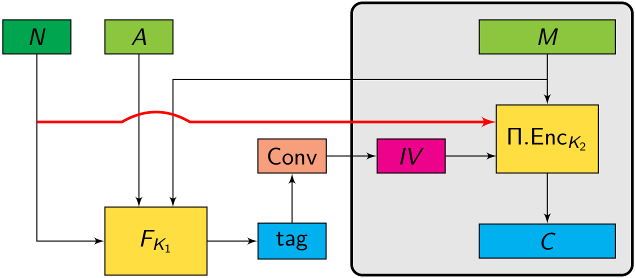

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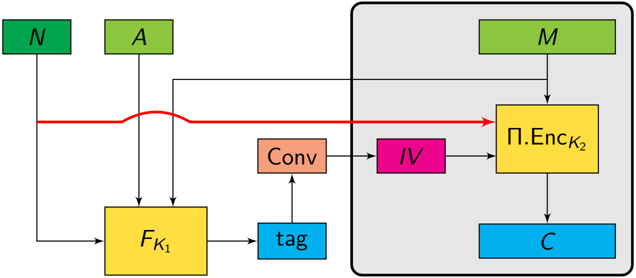

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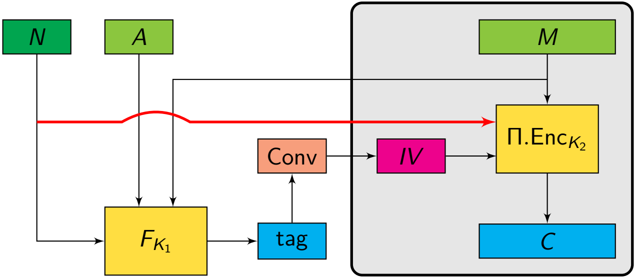

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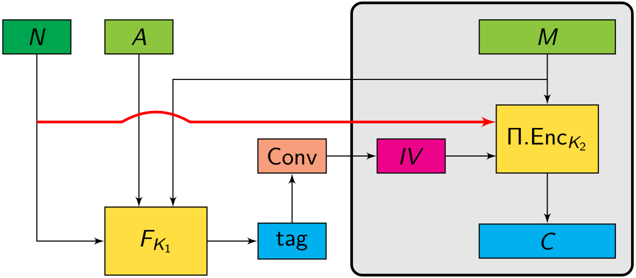

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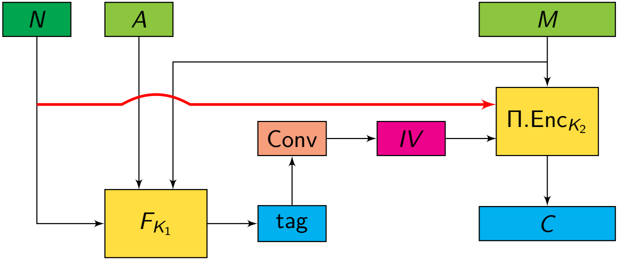
Security Result for NSIV



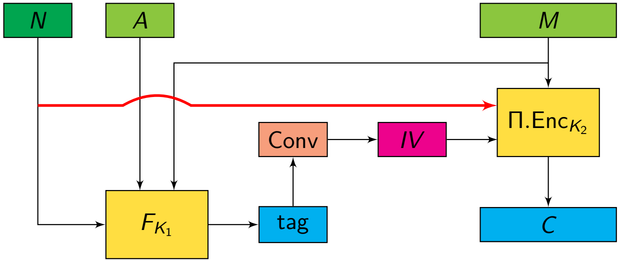
Theorem

For any adversary $\mathcal{A}$ against NSIV$[F,\Pi]$,

$$\mathbf{Adv}^{\mathrm{nAE}}_{\mathsf{NSIV}}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{nivE}}_{\Pi}(\mathcal{A}') + \mathbf{Adv}^{\mathrm{nPRF}}_{F}(\mathcal{A}'') + \mathbf{Adv}^{\mathrm{nMAC}}_{F}(\mathcal{A}''').$$

Moreover, if $\mathcal{A}$ repeats any nonce at most m times, then $\mathcal{A}'$, $\mathcal{A}''$, and $\mathcal{A}'''$ also repeat any nonce at most m times.

Instantiating F and Π



Remaining of the talk:

How to instantiate the PRF F and the nivE encryption scheme Π from a TBC $\widetilde{E}$ so that

- we get BBB-security in the nonce-respecting setting
- we retain birthday-bound security in the nonce-misuse setting

Outline

TBCs and AE

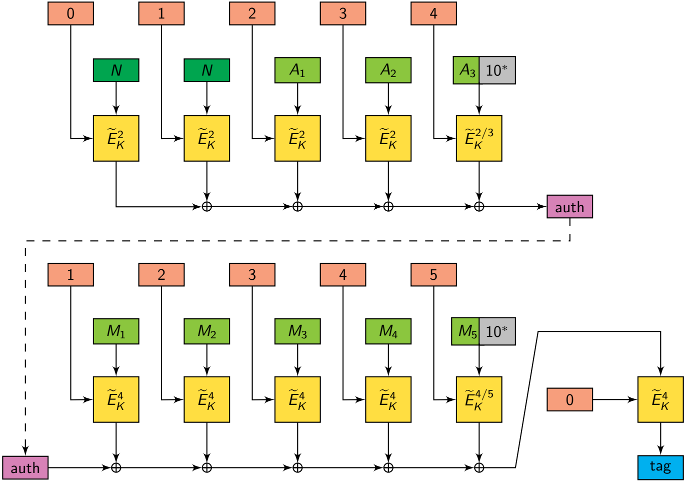
Generic Composition: the NSIV Method

Authentication: the EPWC Mode

Encryption: the CTRT Mode

Conclusion

The EPWC (*Encrypted Parallel Wegman-Carter*) Mode



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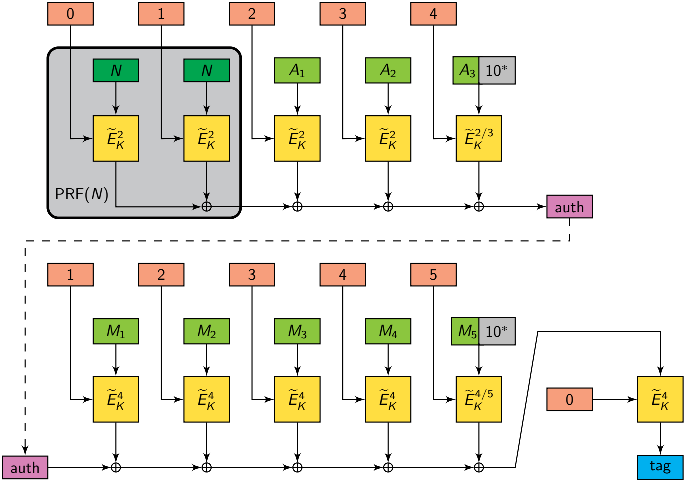

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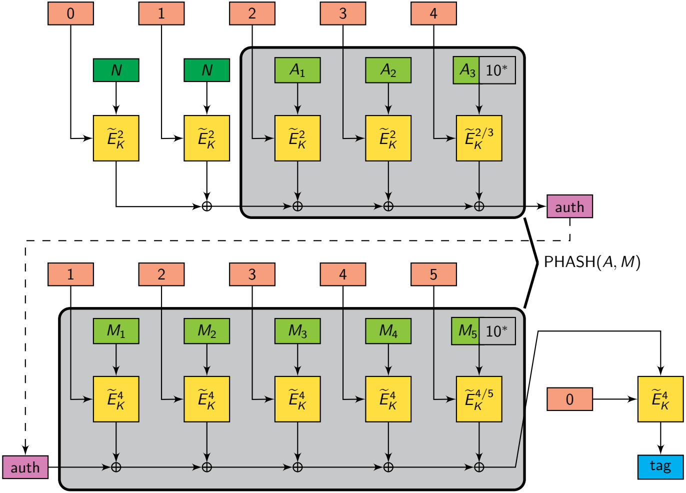

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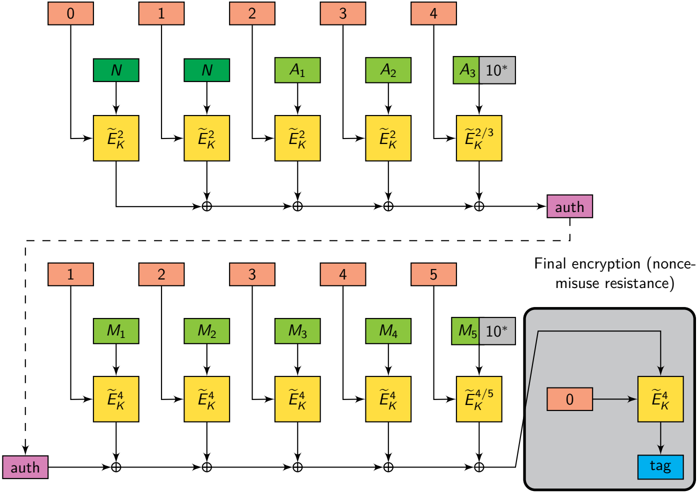

Final encryption (nonce-misuse resistance)

Security of EPWC

Theorem

Let $\mathcal{A}$ be an adversary against EPWC *with an ideal TBC with block-length n making at most q queries. Then*

(a) *If $\mathcal{A}$ is nonce-respecting,*

$$\mathbf{Adv}_{\mathsf{EPWC}}^{\mathrm{nPRF}}(\mathcal{A}) \leq \mathcal{O}\left(\frac{q}{2^n}\right), \qquad \mathbf{Adv}_{\mathsf{EPWC}}^{\mathrm{nMAC}}(\mathcal{A}) \leq \mathcal{O}\left(\frac{q}{2^n}\right).$$

(b) *If $\mathcal{A}$ is allowed to repeat nonces, then*

$$\mathbf{Adv}_{\mathsf{EPWC}}^{\mathrm{PRF}}(\mathcal{A}) \leq \frac{q^2}{2^n}, \qquad \mathbf{Adv}_{\mathsf{EPWC}}^{\mathrm{MAC}}(\mathcal{A}) \leq \frac{q^2+q}{2^n}.$$

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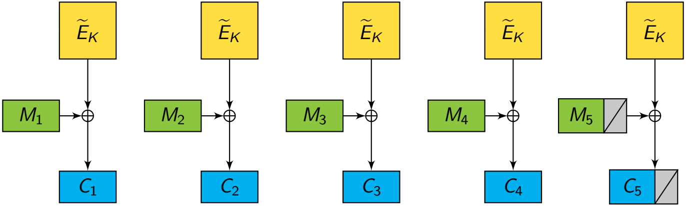

- how to build a counter-like nivE encryption scheme?
- nonce in the tweak $\Rightarrow$ birthday attack!
- switch inputs: nonce in "message input" and counter in tweak
- key observation: $T \mapsto \widetilde{E}_K(T, N)$ is a pseudorandom *function*

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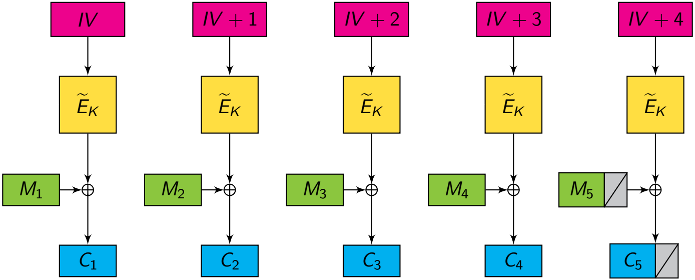

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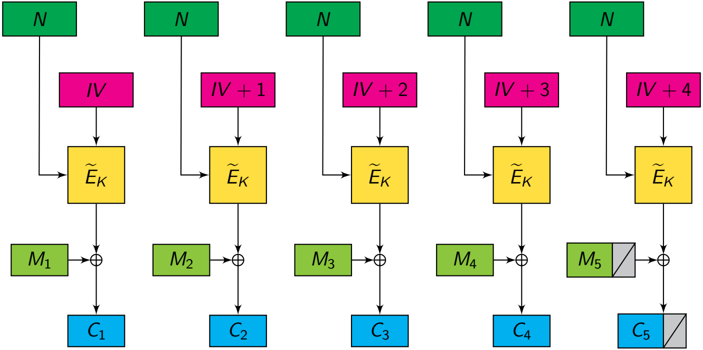

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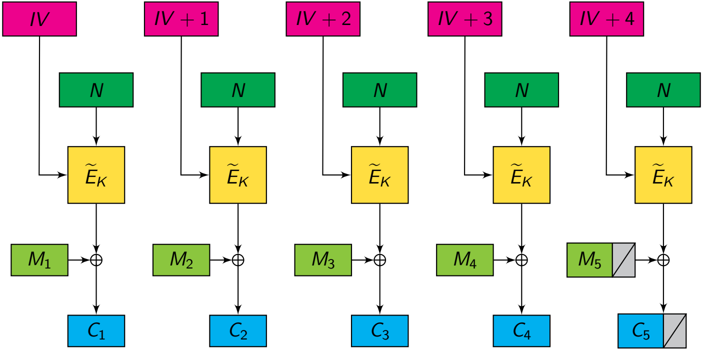

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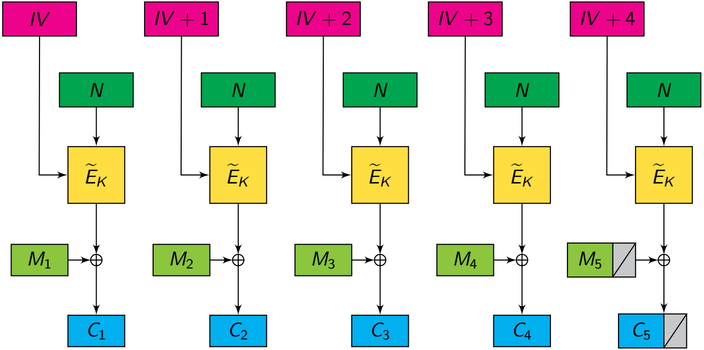

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Security of CTRT

Theorem

- *$n =$ block-length*
- *$t =$ tweak-length*
- *$\sigma =$ total length of queries (in n-bit blocks)*
- *$m =$ maximal number of repetitions of any nonce*

$$\mathbf{Adv}^{\mathrm{nivE}}_{\mathsf{CTRT}}(\mathcal{A}) \le \frac{2(m-1)\sigma}{2^t} + \frac{1}{2^t} + \begin{cases} \dfrac{2\sigma\log^2\sigma}{2^n} & \text{when } \sigma \le 2^t, \\ \dfrac{2t^2\sigma^2}{2^{n+t}} & \text{when } \sigma \ge 2^t. \end{cases}$$

- nonce-respecting ($m = 1$): security up to $\sigma \simeq \min\{2^n, 2^{(n+t)/2}\}$
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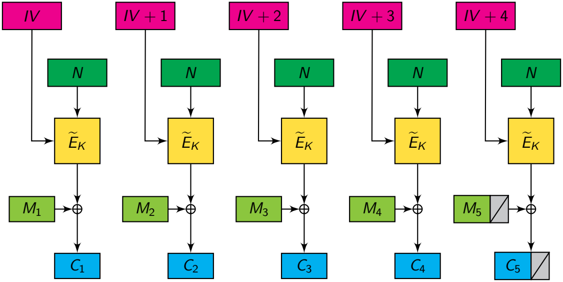

- assume first that nonces are never repeated
- we want to show that ciphertexts are indist. from random
- each random IV determines the sequence of tweaks $(IV, IV+1, \ldots)$ used in the TBC
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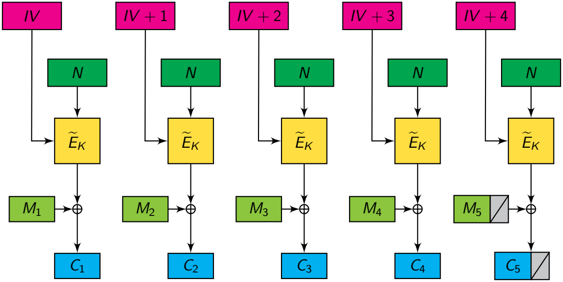

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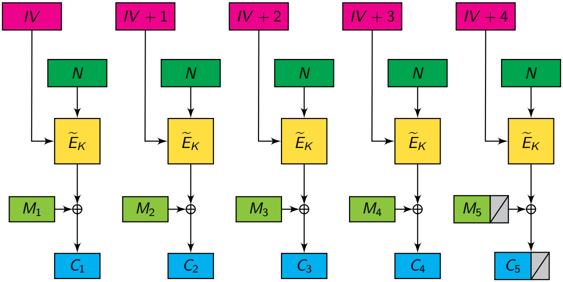

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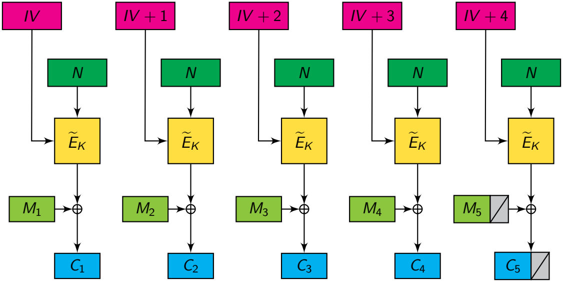

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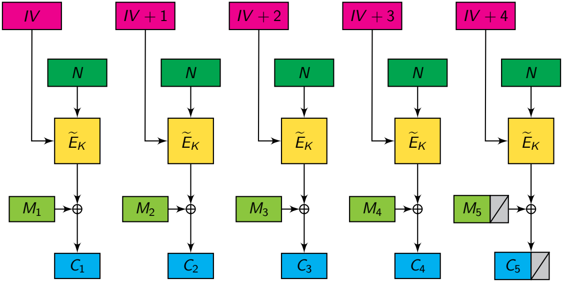

- for each tweak, we have an independent PRF/PRP distinguishing problem with $L(T)$ "queries" (nonces):

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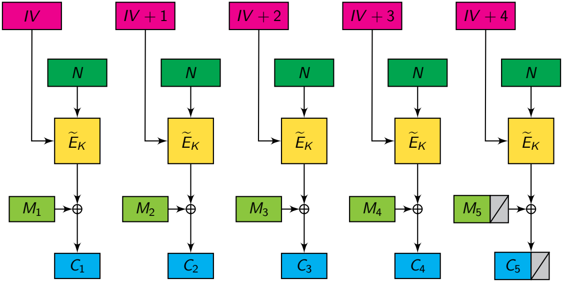

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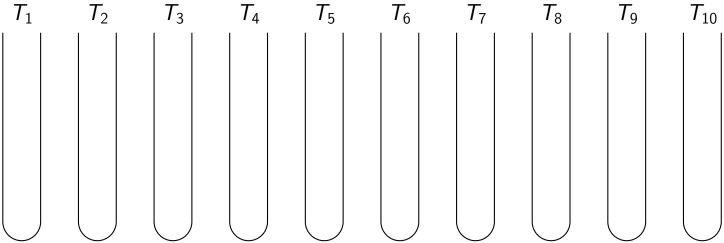

- 2^t bins = tweak values
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- except with probability $1/2^t$, one has
  (a) if $\sigma \le 2^t$, then $\max L(T) \le 2 \log \sigma$;
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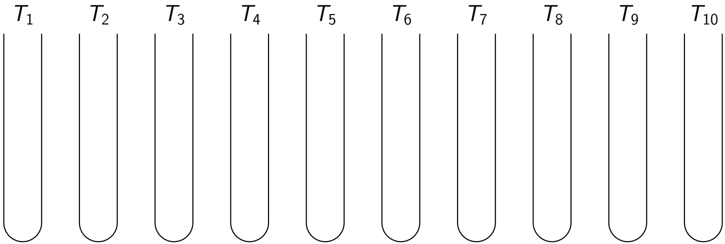

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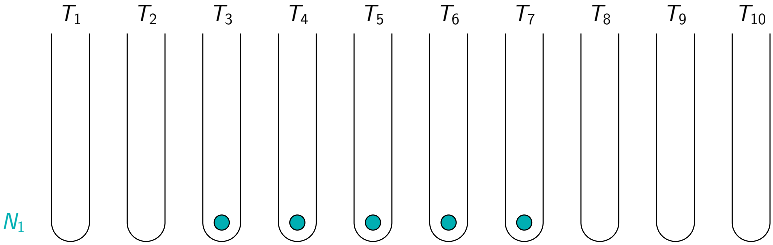

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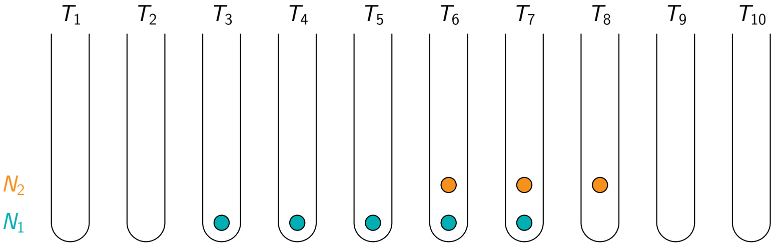

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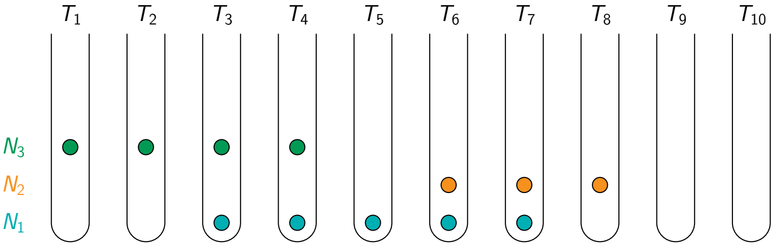

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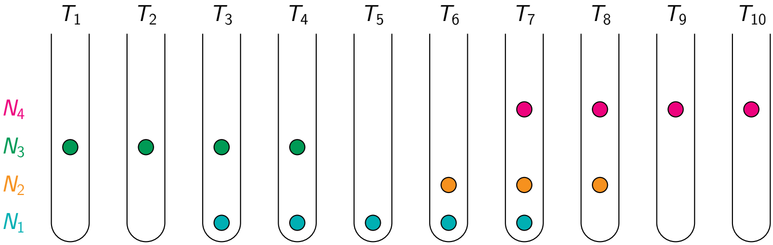

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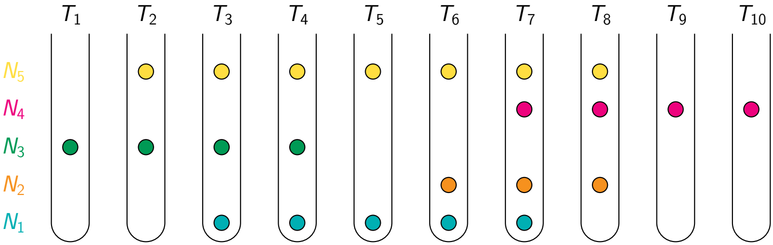

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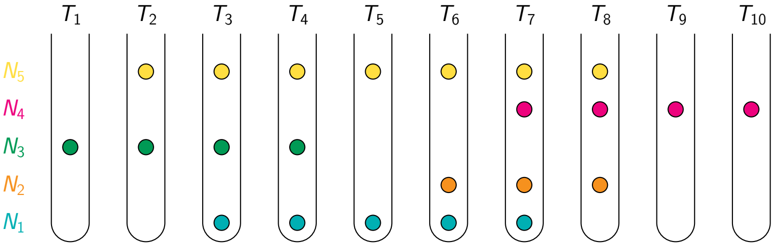

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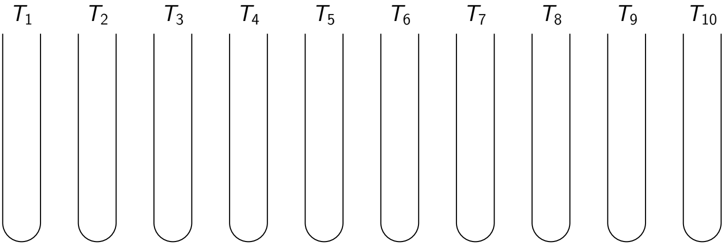
Proof of Security of CTRT (nonce-misuse)



- bad event that allows to distinguish outputs from random: $\exists$ two encryption queries with the same nonce and a common tweak (counter)
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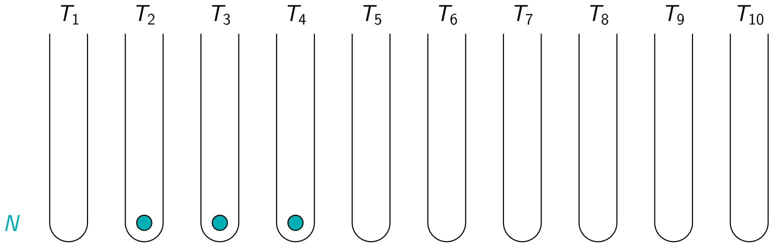

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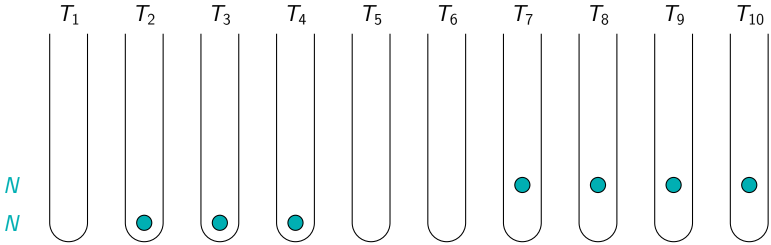

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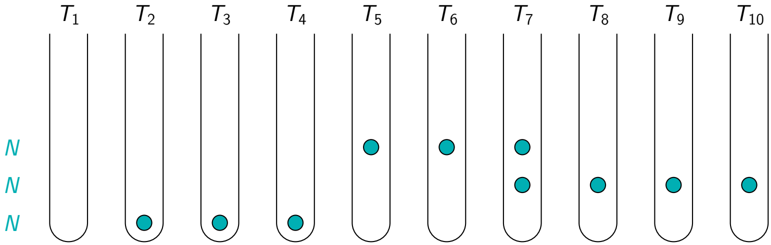

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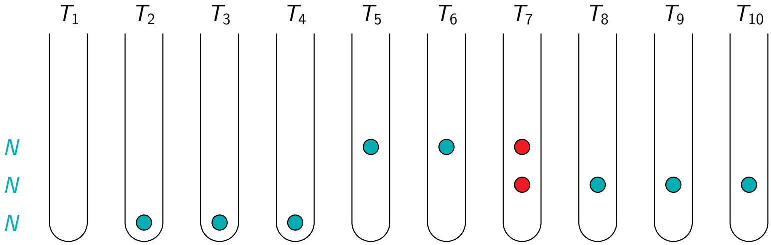

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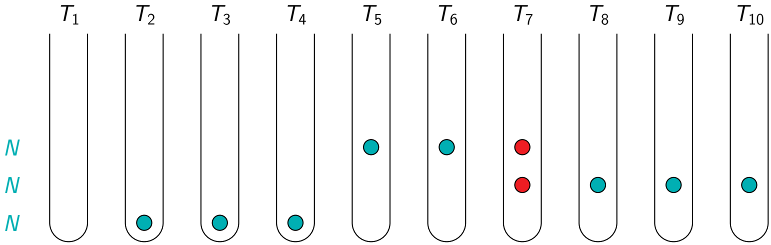

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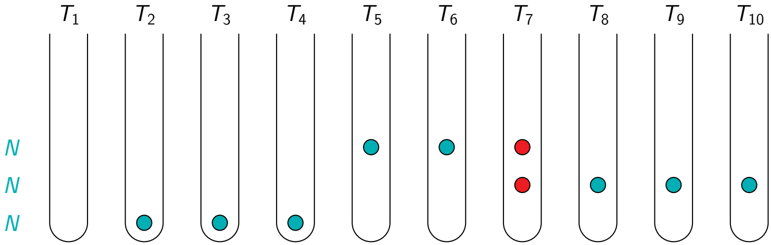

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Outline

TBCs and AE

Generic Composition: the NSIV Method

Authentication: the EPWC Mode

Encryption: the CTRT Mode

Conclusion

Wrap-up and Final Remarks

- EPWC + CTRT combined using the NSIV composition method
  = SCT (*Synthetic Counter in Tweak*) mode
- BBB-secure in the nonce-respecting setting
- retains birthday-bound security in the nonce-misuse setting
- parallel, quite efficient, does not need the decryption direction
- instantiation of the TBC: needs to be BBB-secure!
  ⇒ XEX does not work
  ⇒ use ad-hoc TBCs such as Deoxys-BC and Joltik-BC

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The end. . .

Thanks for your attention!

Comments or questions?

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