Beyond-Birthday-Bound Secure MACs

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Introduction

- we survey recent results on MAC constructions which are
  - based on a block cipher (BC) or a tweakable block cipher (TBC)
  - secure beyond the birthday bound (BBB-secure)
- most (T)BC-based MACs are secure only up to the birthday-bound w.r.t. to the block size n: they become insecure when $\sim 2^{n/2}$ (blocks of) messages have been treated
- BBB-security is important for lightweight crypto (small blocks, inconvenient re-keying,...)
- we highlight some open problems along the way

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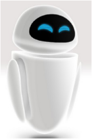
Generalities

Stateless Deterministic MACs
- The UHF-then-PRF Paradigm
- Constructing BBB-Secure Output Functions from (T)BCs
- Constructing BBB-Secure UHFs from (T)BCs

Nonce-Based MACs
- State of Art
- Open Problems

MAC Definition



$T = \mathrm{MAC}_K(N, M)$

$\mathrm{MAC}_K(N', M') = T'$?

Security Definition

The adversary is allowed

- q MAC queries $T = \mathrm{MAC}_K(N, M)$
- v verification queries (forgery attempts) (N', M', T')

and is successful if one of the verification queries (N', M', T') passes and no previous MAC query (N', M') returned T'.

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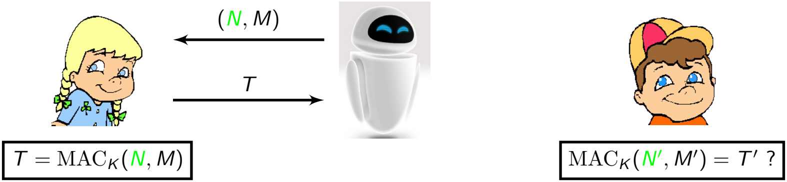

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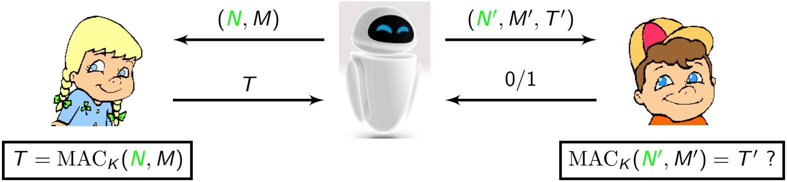

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Three types of MAC

- stateless and deterministic: MAC function only takes the key and the message as input
  (Variable-input-length PRF $\Rightarrow$ stateless deterministic MAC)
- nonce-based:
  - MAC function takes as input a non-repeating nonce N in addition to the key and the message M
  - sec. model: the nonce is chosen by the adversary
  - the adversary is said nonce-respecting if it does not repeat nonces in MAC queries and nonce-misusing otherwise
- randomized: MAC function takes as input random coins R (generated by the sender) in addition to the key and the message

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Graceful Nonce-Misuse Security Degradation

- the security of some nonce-based MACs collapses if a single nonce is repeated (e.g. GMAC)
- ideally, security should degrade gracefully in case nonces are repeated
- any BBB-secure nonce-based MAC with graceful security degradation can be turned into a BBB-secure randomized MAC by choosing n-bit nonces uniformly at random:

$$\mathbf{Adv}_F^{\text{rand-MAC}}(q,v) \le \underbrace{\frac{q^{\mu+1}}{2^{\mu(n+1)}}}_{\substack{\mu\text{-multicoll.}\\ \text{proba.}}} + \mathbf{Adv}_F^{\text{nonce-MAC}}(q,v,\mu)$$

where μ is the maximal number of nonce repetitions.

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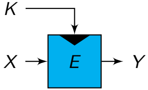
Building Blocks: BCs and TBCs



n = block size
t = tweak size

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- tweakable block cipher $\widetilde{E}$: for each key K and each tweak W, $X \mapsto \widetilde{E}(K, W, X)$ is a permutation
- one can think of a keyed TBC $\widetilde{E}_K$ as an "imperfect" $(n+t)$-to-n-bit PRF
- if any tweak W is used at most "a few" times, $\widetilde{E}_K$ is close to a random $(n+t)$-to-n-bit function

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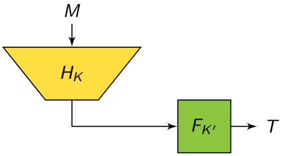

- based on a fixed-input-length PRF F and an ε-almost universal (ε-AU) hash function H:

$$\forall M \neq M', \; \Pr[K \leftarrow_{\$} \mathcal{K} : H_K(M) = H_K(M')] \leq \varepsilon$$

- H can be statistically secure (polynomial evaluation) or computationally secure (BC/TBC-based)
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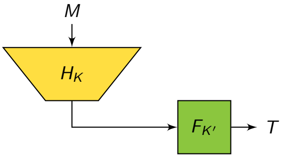

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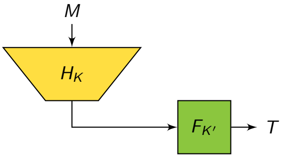

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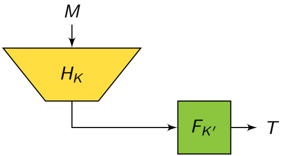
Security of UHF-then-PRF



- birthday-bound-secure w.r.t. H collision probability ε

$$\mathbf{Adv}^{\mathrm{PRF}}_{F \circ H}(q) \leq \frac{q^2 \varepsilon}{2} + \mathbf{Adv}^{\mathrm{PRF}}_{F}(q)$$

- typical instantiation from a block cipher E:
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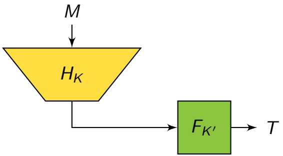

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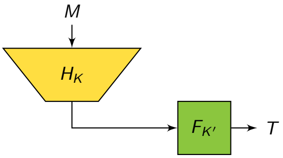

- for BBB-security, we need a $2n$-bit output UHF with $\varepsilon \simeq 2^{-2n}$ and a BBB-secure $2n$-to-n-bit PRF
- constructing a BBB-secure $2n$-to-n-bit PRF from an n-bit block cipher seems inconvenient
(e.g. XOR2 construction [Luc00, Pat08, DHT17] + 5-round Feistel [Pat04])
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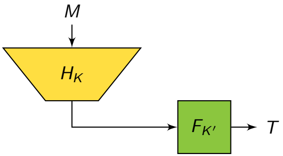

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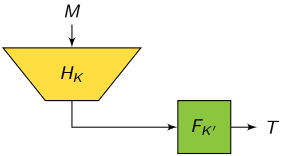

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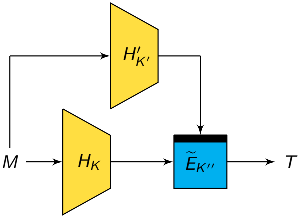
TBC-Based Constructions [CLS17, LN17]



Hash as Tweak (HaT) [CLS17]

Hash-then-TBC [LN17]

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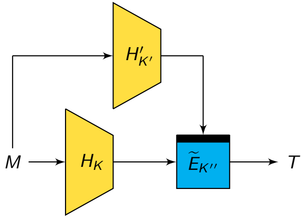

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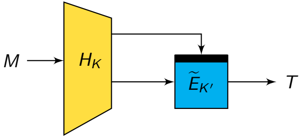

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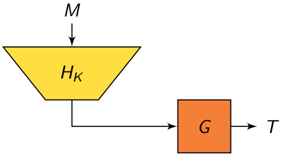

- the output function need not be keyed
- modeling G as a RO, the construction is secure if H is ε-AU and ε'-uniform:

$$\forall M,\ \forall Y,\ \Pr[K \leftarrow_{\$} \mathcal{K} : H_K(M) = Y] \leq \varepsilon'$$

- security proof under a standard assumption on G?

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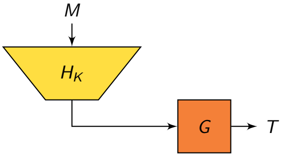

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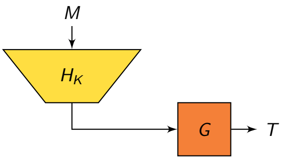

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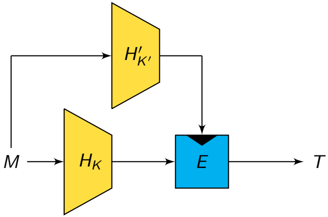
BBB-Secure Instantiation from an Ideal BC [CLS17]



Hash as Key (HaK)

- the HaK construction is BBB-secure in the ideal cipher model assuming H and H' are ε-AU and ε'-uniform

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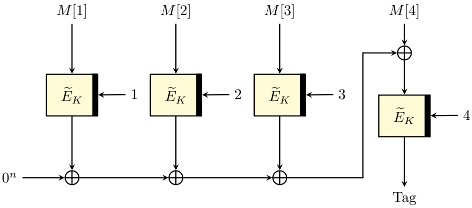
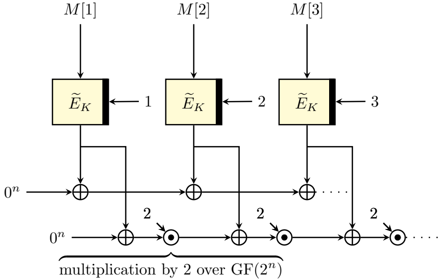
PMAC/PMAC1 [BR02, Rog04]



- most existing constructions are variants of PMAC [BR02] (BC-based) and PMAC1 [Rog04] (TBC-based)
- the underlying hash function (omitting final $\widetilde{E}$ call) is ε-AU for $\varepsilon \simeq 2^{-n}$

PMAC_TBC [Nai15]



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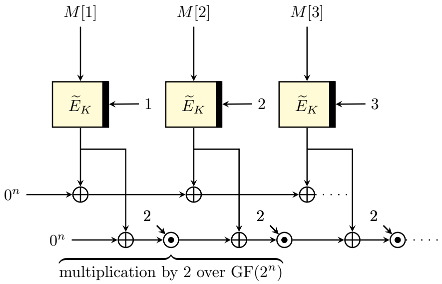

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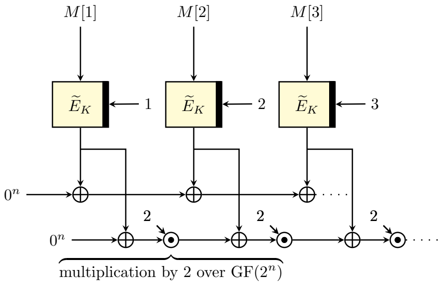

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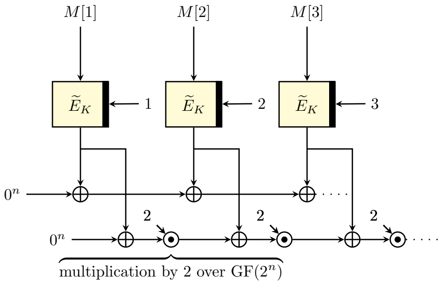

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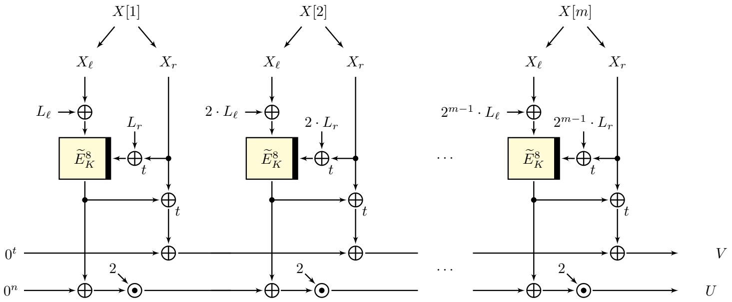
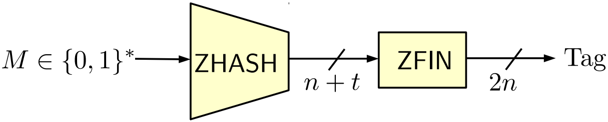
ZHASH [IMPS17]



- each TBC call processes $(n + t)$ bits of message
- uses a variant of the XTX construction [MI15] to extend the tweak space and incorporate the block counter
- ZHASH is ε-AU for $\varepsilon = 4/2^{n+\min\{n,t\}}$

ZMAC [IMPS17] and ZMAC+ [LN17]



- ZMAC [IMPS17] combines ZHASH and an $(n + t)$-to-n-bit PRF constructed from the TBC using the UHF-then-PRF paradigm
- ZMAC+ [LN17] improves the efficiency of the output function using the Hash-then-TBC construction

Open Problems

- alternative to UHF-then-PRF:
  - finalization function in PMAC_Plus: $(U, V) \mapsto E_{K_1}(U) \oplus E_{K_2}(V)$ $\Rightarrow$ not a PRF
  - find a generic composition theorem capturing the security proofs of PMAC_Plus and PMAC_TBC
- exact security of PMAC_Plus?
- efficient BC-based constructions with n-bit security? (F_t construction [IM16] and LightMAC_Plus2 [Nai17] achieve $kn/(k+1)$-bit security with a kn bit state)

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- exact security of PMAC_Plus?
- efficient BC-based constructions with n-bit security? (F_t construction [IM16] and LightMAC_Plus2 [Nai17] achieve $kn/(k+1)$-bit security with a kn bit state)

Open Problems

- alternative to UHF-then-PRF:
  - finalization function in PMAC_Plus: $(U, V) \mapsto E_{K_1}(U) \oplus E_{K_2}(V)$ $\Rightarrow$ not a PRF
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Outline

Generalities

Stateless Deterministic MACs

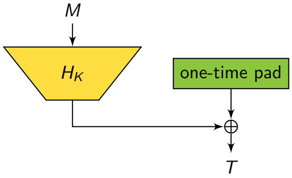
- The UHF-then-PRF Paradigm
- Constructing BBB-Secure Output Functions from (T)BCs
- Constructing BBB-Secure UHFs from (T)BCs

Nonce-Based MACs

- State of Art
- Open Problems

Outline

The Wegman-Carter Construction [GMS74, WC81]



- based on an ε-almost xor-universal (ε-AXU) hash function H:

$$\forall M \neq M',\ \forall Y,\ \Pr[K \leftarrow_{\$} \mathcal{K} : H_K(M) \oplus H_K(M') = Y] \leq \varepsilon$$

- in practice, OTPs are replaced by a PRF applied to a nonce N
- H usually based on polynomial evaluation (GCM, Poly1305)
- "optimal" security:

$$\mathbf{Adv}^{\mathrm{MAC}}_{\mathrm{WC}}(q, v) \leq v\varepsilon + \mathbf{Adv}^{\mathrm{PRF}}_{F}(q + v)$$

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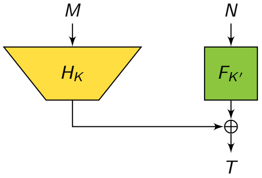

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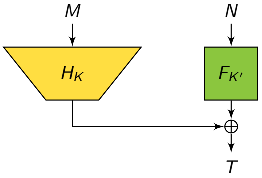

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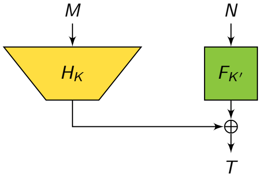

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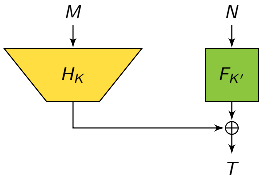
Implementing the PRF from a Block Cipher



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- provable security drops to birthday bound ☹ [Sho96]

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- a better bound exists [Ber05] but still "birthday-type"
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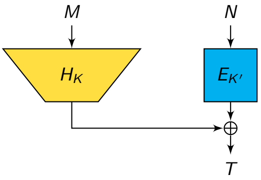

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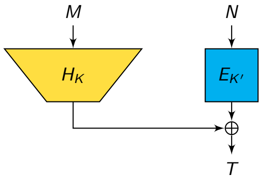

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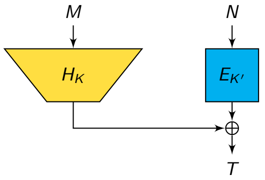

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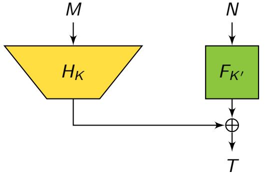

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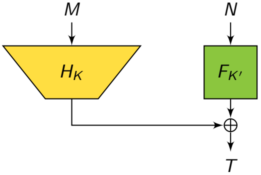

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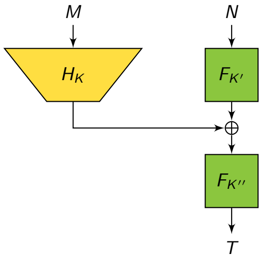

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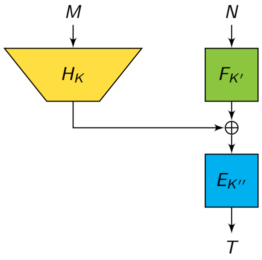

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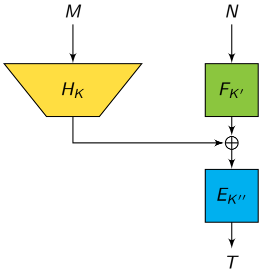

- what if we instantiate $F_{K'}$ with the Davies-Meyer construction $\mathrm{DM}[E]_{K'}(N) = E_{K'}(N) \oplus N$?
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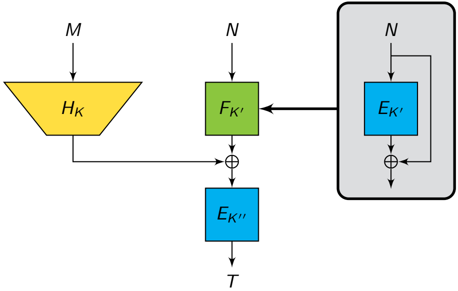

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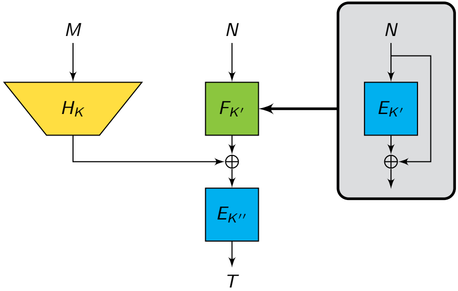

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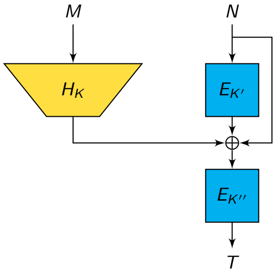

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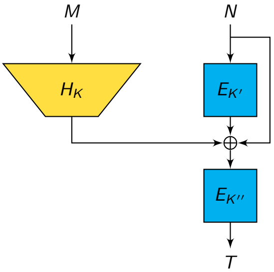

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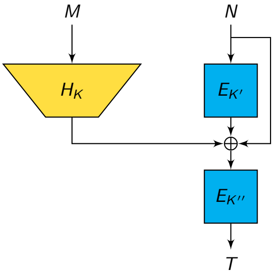

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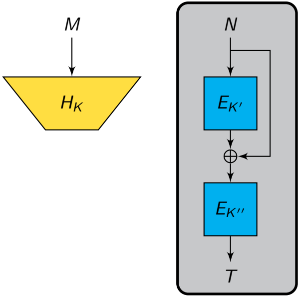

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EDM is a secure PRF up to:

- $2^{2n/3}$ queries (H-coefficients) [CS16]
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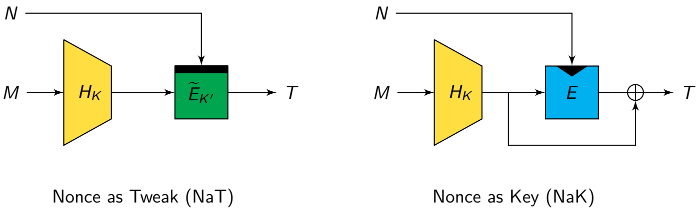
TBC and IC-Based Finalization [CLS17]



- both constructions enjoy graceful security degradation with maximal nonce multiplicity μ

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- NaK construction provably secure in the ideal cipher model, assuming H is ε-AXU and uniform (Davies-Meyer mode required to make the output function non-invertible!)

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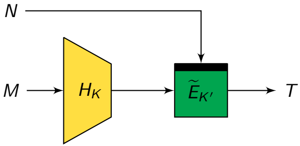

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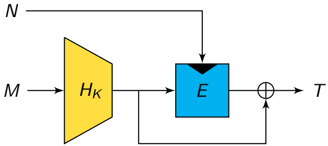

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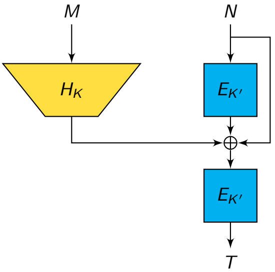
Optimizing and Instantiating EWCDM



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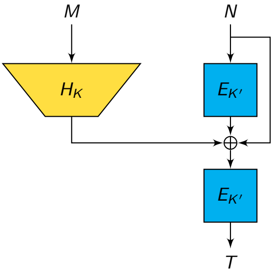

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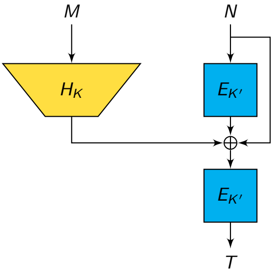

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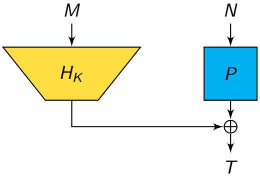

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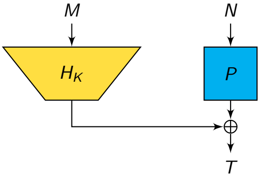

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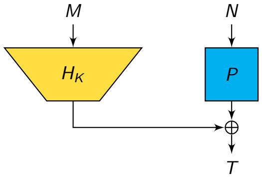
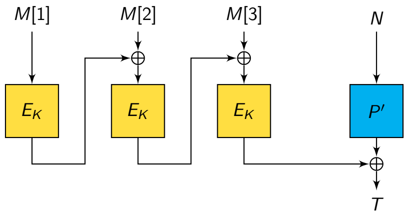


- security bound (one forgery attempt):

$$\mathbf{Adv}_{\mathrm{WC}}^{\mathrm{MAC}}(q,1) \leq v\varepsilon + \frac{(q+1)^2}{2 \cdot 2^n}$$

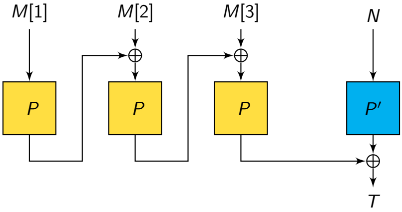
- matching attack when $H_K(M) = K \cdot M$:
  - make $q \sim 2^{n/2}$ MAC queries $(N_i, M_i) \to T_i$
  - for each pair (i,j), $K \cdot (M_i \oplus M_j) \neq T_i \oplus T_j$
  - $\Rightarrow$ discard $\sim 2^n$ bad keys
- security bound is tight (number of queries)

WCS with a Computational BC-based UHF



- instantiate H_K with e.g. CBC$[E_K]$
- replace E_K by a random permutation P (PRP term)
  $\Rightarrow$ previous information-theoretic attack does not work anymore
- very similar to CCM authentication
  $\rightarrow$ conjectured BBB-secure by Jonsson [Jon02]

WCS with a Computational BC-based UHF



- instantiate H_K with e.g. CBC$[E_K]$
- replace E_K by a random permutation P (PRP term)
  $\Rightarrow$ previous information-theoretic attack does not work anymore
- very similar to CCM authentication
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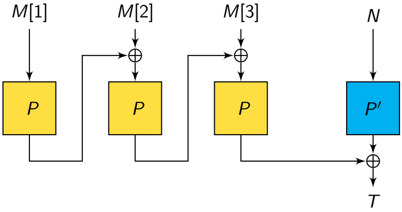

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  $\rightarrow$ conjectured BBB-secure by Jonsson [Jon02]

The end. . .

Thanks for your attention!

Comments or questions?

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