

On the Provable Security of the Iterated Even-Mansour Cipher against Related-Key and Chosen-Key Attacks

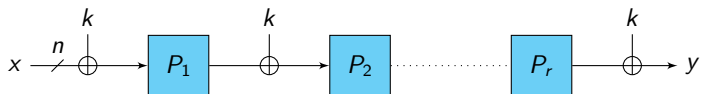
Benoît Cogliati¹ and Yannick Seurin²

¹Versailles University, France

²ANSSI, France

April 16, 2015 — ENS Paris

One-Slide Digest



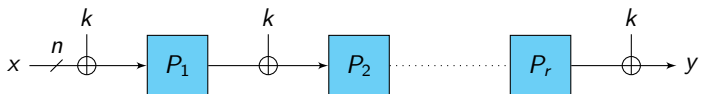
1 round: PRP

3 rounds: XOR-Related-Key-Attacks PRP

4 rounds: Chosen-Key-Attacks Resistance

12 rounds: Full indistinguishability from an ideal cipher

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Outline

Introduction: Key-Alternating Ciphers in the Random Permutation Model

Security Against Related-Key Attacks

Security Against Chosen-Key Attacks

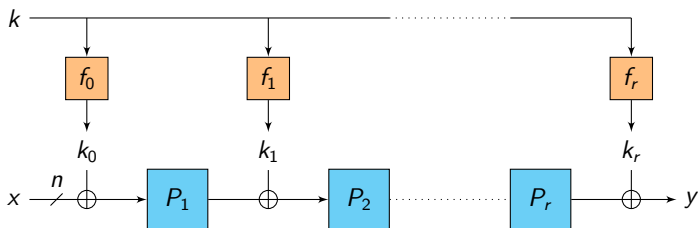
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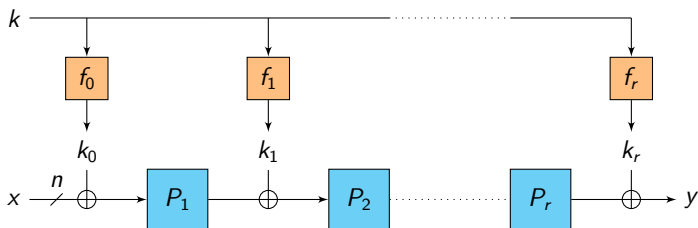
Key-Alternating Cipher (KAC): Definition



An r -round key-alternating cipher:

- plaintext $x \in \{0, 1\}^n$, ciphertext $y \in \{0, 1\}^n$
- master key $k \in \{0, 1\}^\kappa$
- the P_i 's are **public** permutations on $\{0, 1\}^n$
- the f_i 's are key derivation functions mapping k to n -bit “round keys”
- examples: most **SPNs** (AES, SERPENT, PRESENT, LED, ...)

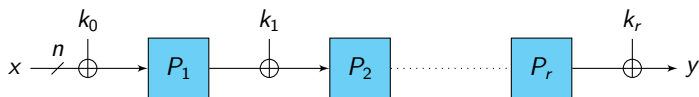
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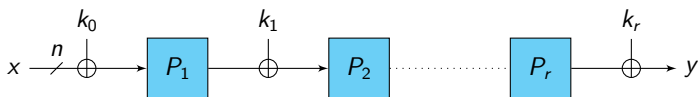
Various Key-Schedule Types



Round keys can be:

- **independent** (total key-length $\kappa = (r + 1)n$)
- derived from an n -bit master key ($\kappa = n$), e.g.
 - **trivial** key-schedule: (k, k, \dots, k)
 - **more complex**: $(f_0(k), f_1(k), \dots, f_r(k))$
- anything else (e.g. $2n$ -bit master key (k_0, k_1) and round keys $(k_0, k_1, k_0, k_1, \dots)$ as in LED-128)

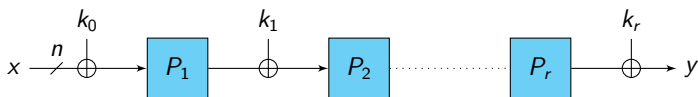
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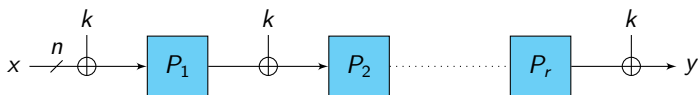
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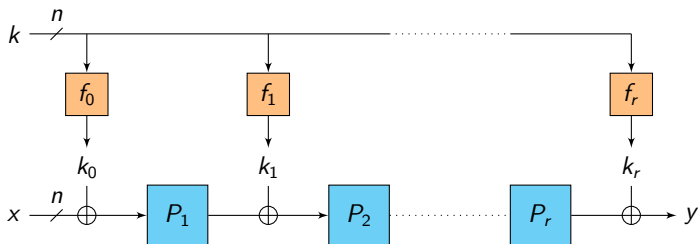
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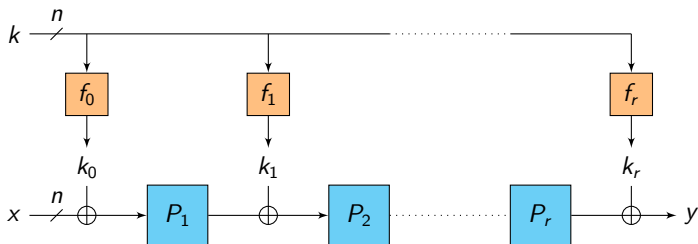
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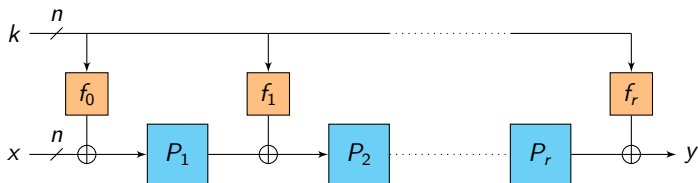
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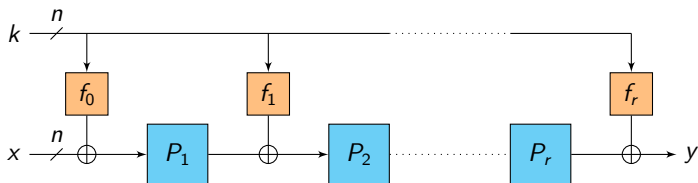


Question

How can we “prove” security?

- against a **general adversary**:
 \Rightarrow too hard (unconditional complexity lower bound!)
- against **specific attacks** (differential, linear...):
 \Rightarrow use specific design of P_1, \dots, P_r (count active S-boxes, etc.)
- against **generic attacks**:
 \Rightarrow Random Permutation Model for P_1, \dots, P_r

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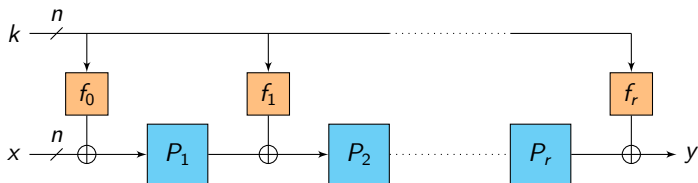


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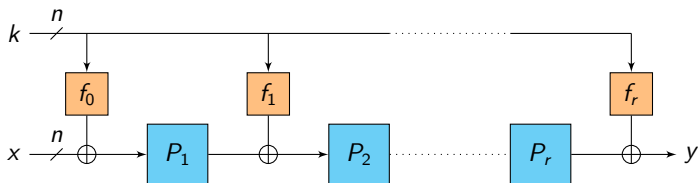


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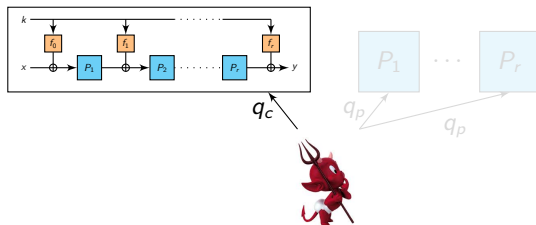


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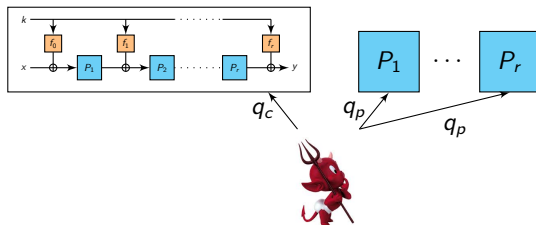
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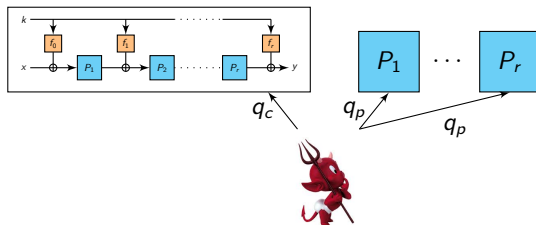
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- trades complexity for randomness (\simeq Random Oracle Model)
- complexity measure of the adversary:
 - $q_c = \#$ queries to the cipher = plaintext/ciphertext pairs (**data** D)
 - $q_p = \#$ queries to each internal permutation oracle (**time** T)
 - but otherwise **computationally unbounded**
- \Rightarrow **information-theoretic** proof of security

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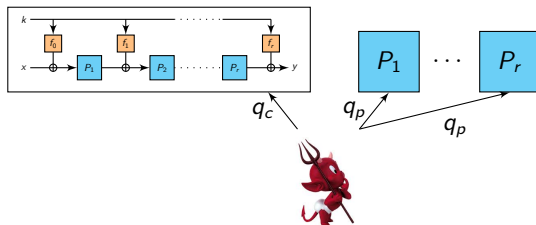
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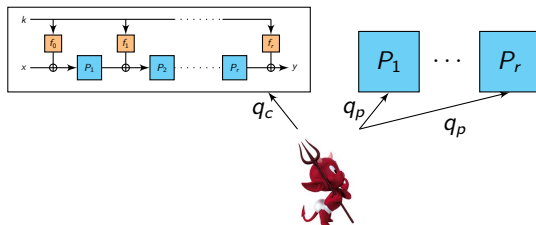
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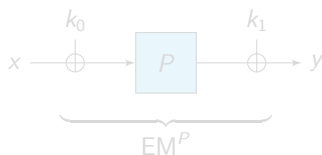


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Analyzing KACs in the Random Permutation Model

Even and Mansour seminal work:

- this model was first proposed by **Even and Mansour** at ASIACRYPT '91 for $r = 1$ round
- they showed that the simple cipher $k_1 \oplus P(k_0 \oplus x)$ is a secure PRP up to $\sim 2^{\frac{n}{2}}$ queries of the adversary to P and to the cipher
- similar result when $k_0 = k_1$ [KR01, DKS12]

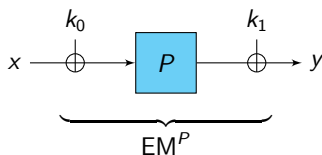


- improved bound as r increases: PRP up to $\sim 2^{\frac{m}{r+1}}$ queries [CS14]

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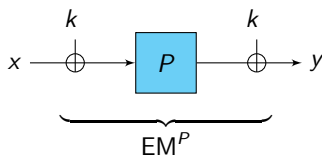


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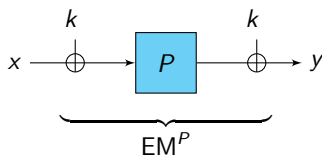


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A Word on Wording

“the” Iterated Even-Mansour (IEM) Cipher

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generic class of key-alternating ciphers
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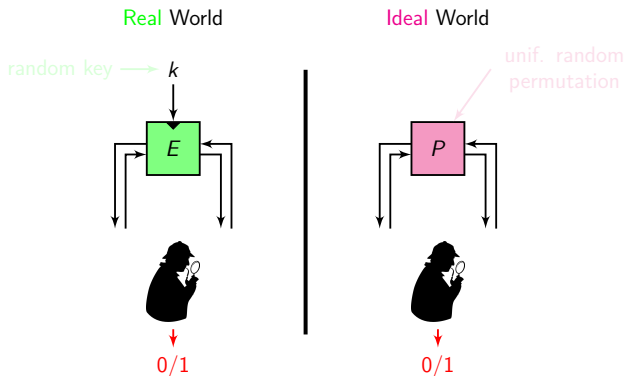
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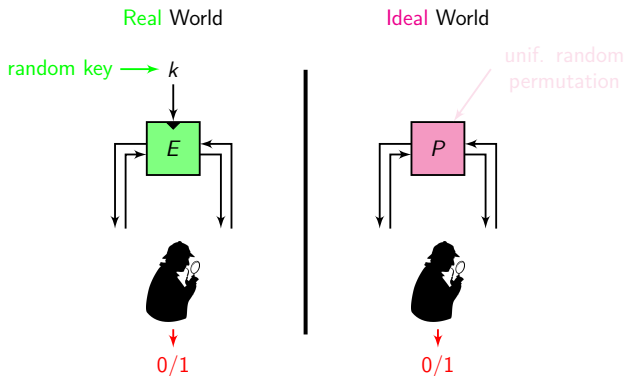
Formalizing Block Cipher Security: Pseudorandomness



SPRP (a.k.a. CCA) advantage:

$$\text{Adv}_E^{\text{SPRP}}(\mathcal{D}) = \left| \Pr \left[\mathcal{D}^{E_k} = 1 \right] - \Pr \left[\mathcal{D}^P = 1 \right] \right|$$

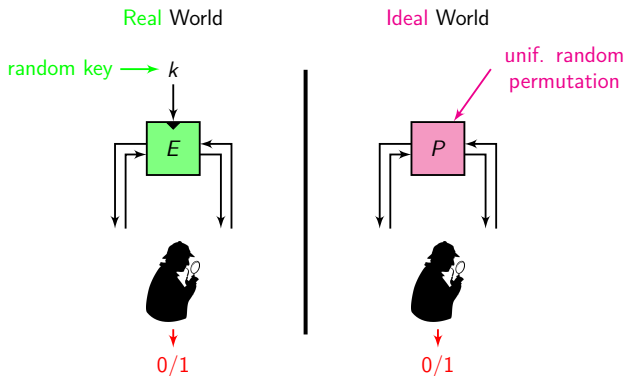
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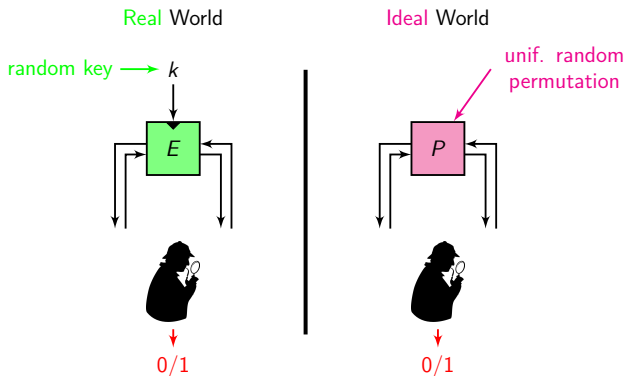
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Related-Key Attacks

The Related-Key Attack Model [BK03]:

- stronger adversarial model: the adversary can specify **Related-Key Deriving (RKD) functions** ϕ and receive $E_{\phi(k)}(x)$ and/or $E_{\phi(k)}^{-1}(y)$
- the block cipher should behave as an **ideal cipher** (an independent random permutation for each key)
- **impossibility results** for too “large” sets of RKDs
- positive results for **limited sets of RKDs** or using **number-theoretic constructions**
- we will consider **XOR-RKAs**: the set of RKD functions is

$$\{\phi_{\Delta} : k \mapsto k \oplus \Delta, \Delta \in \{0, 1\}^k\}$$

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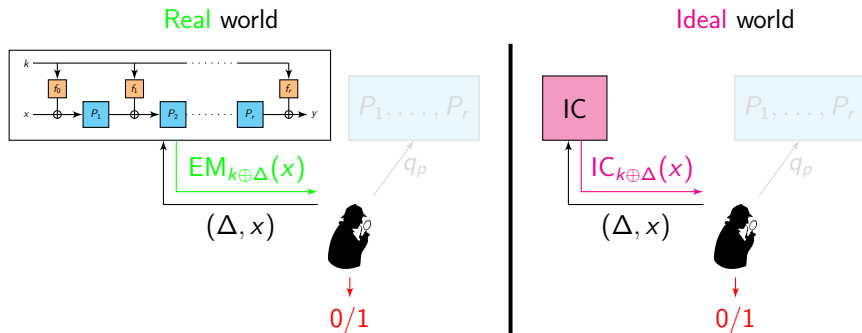
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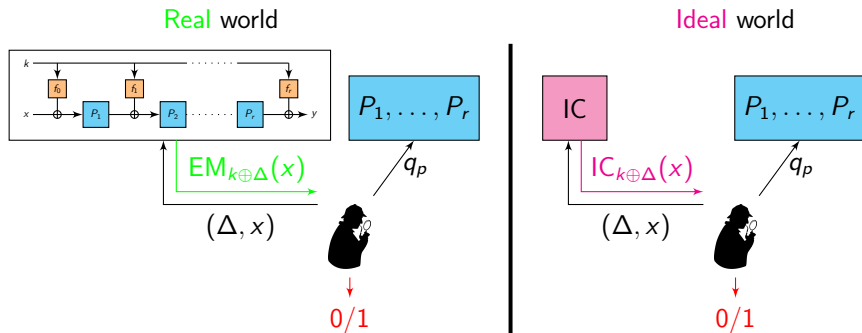
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XOR-RKAs against the IEM Cipher: Formalization



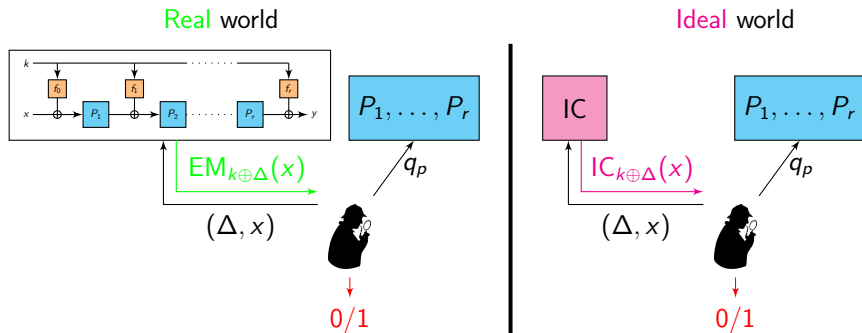
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- **ideal** world: ideal cipher IC independent from P_1, \dots, P_r
- Rand. Perm. Model: \mathcal{D} has oracle access to P_1, \dots, P_r in both worlds
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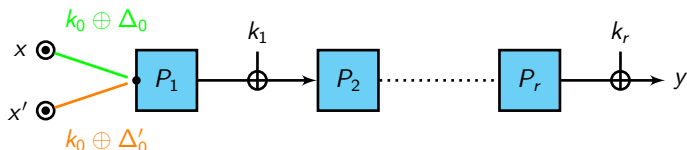
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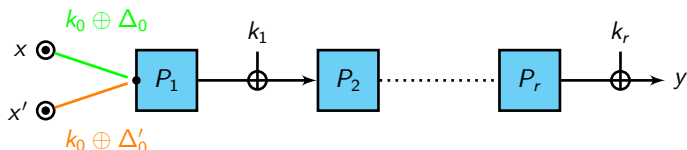
RK Distinguisher for independent round keys:

- query $((\Delta_0, 0, \dots, 0), x)$ and $((\Delta'_0, 0, \dots, 0), x')$ such that

$$x \oplus \Delta_0 = x' \oplus \Delta'_0$$

- check that the outputs are equal
- holds with proba. 1 for the IEM cipher
- holds with proba. 2^{-n} for an ideal cipher
- \Rightarrow we will consider “dependent” round keys (in part. (k, k, \dots, k))

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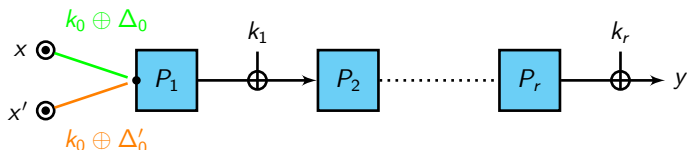
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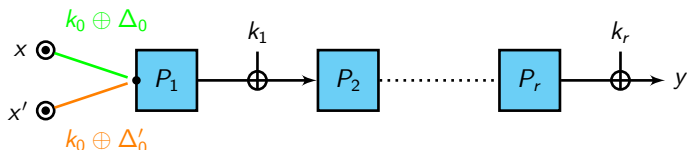
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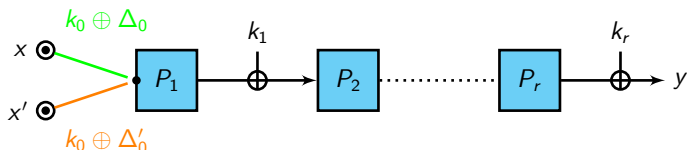
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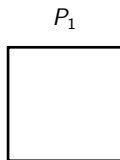
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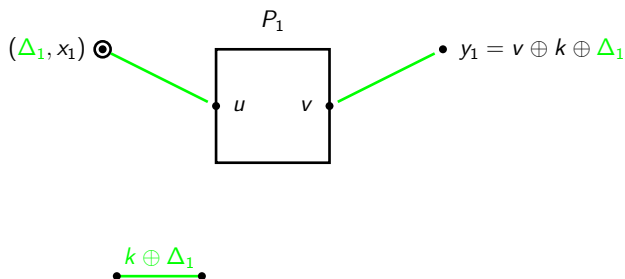
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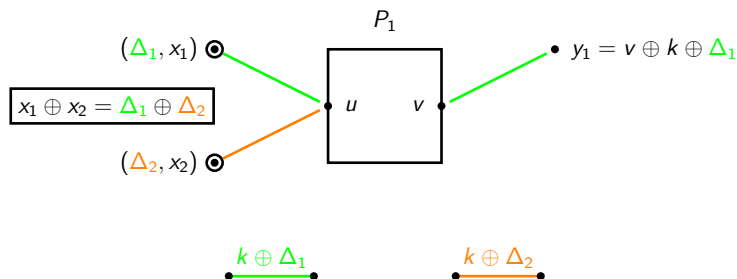
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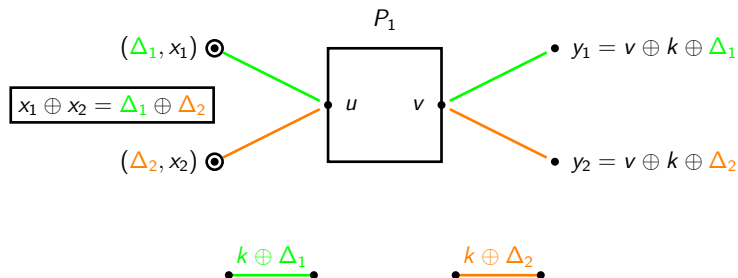
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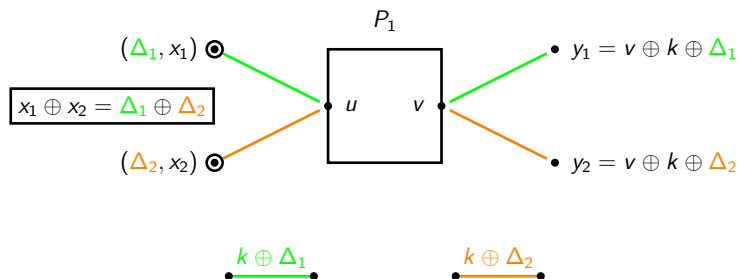
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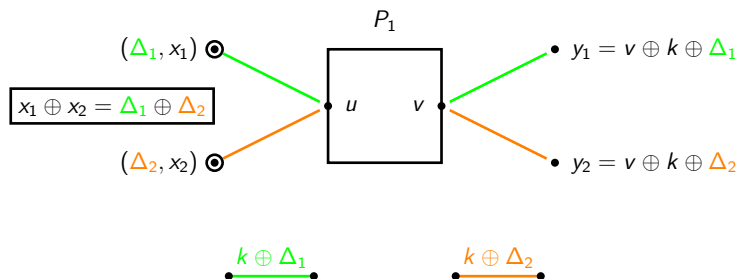
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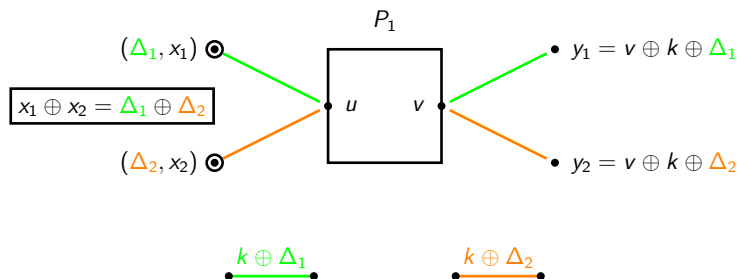
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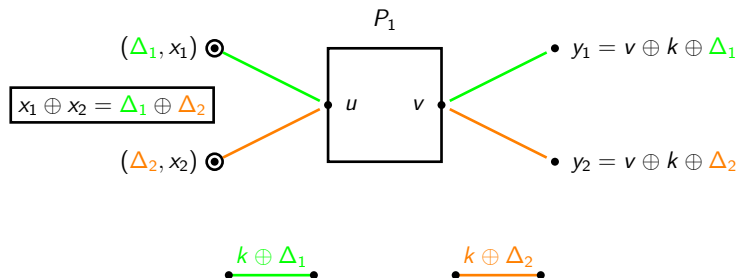
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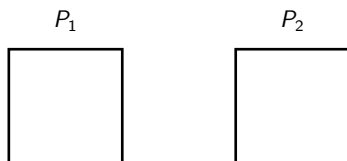
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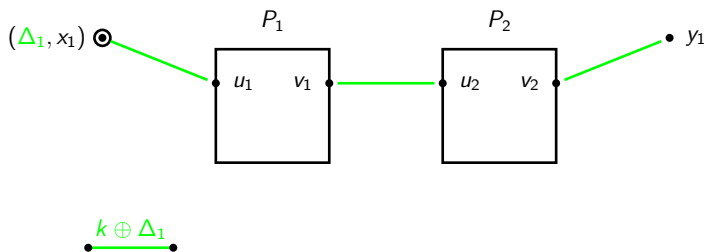
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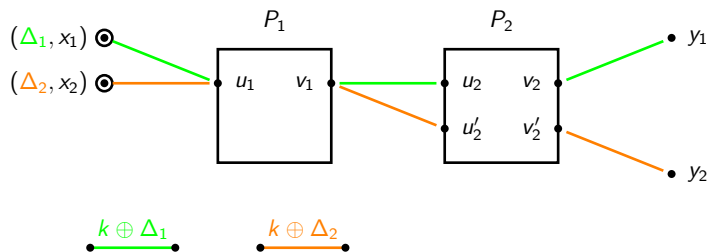
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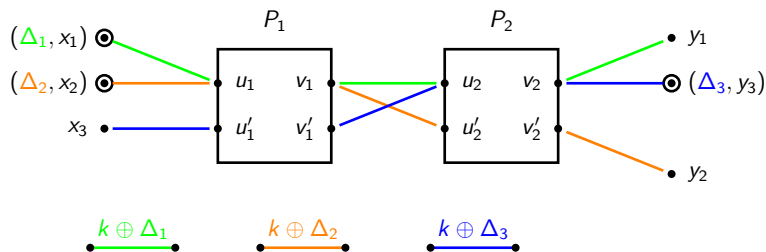
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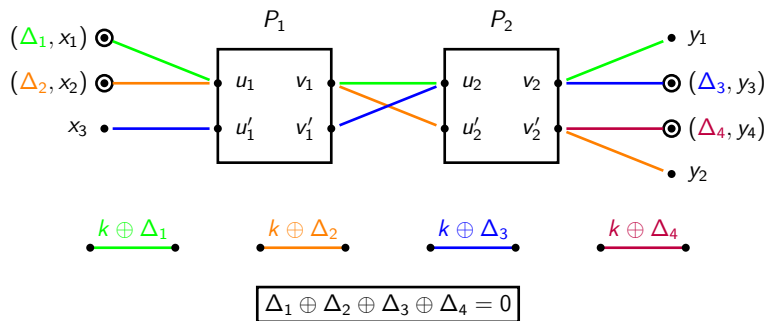
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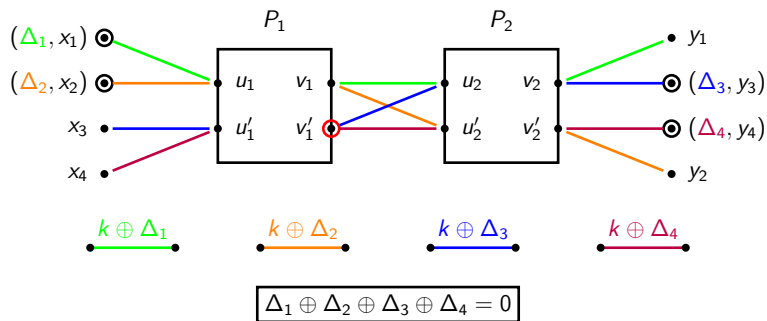
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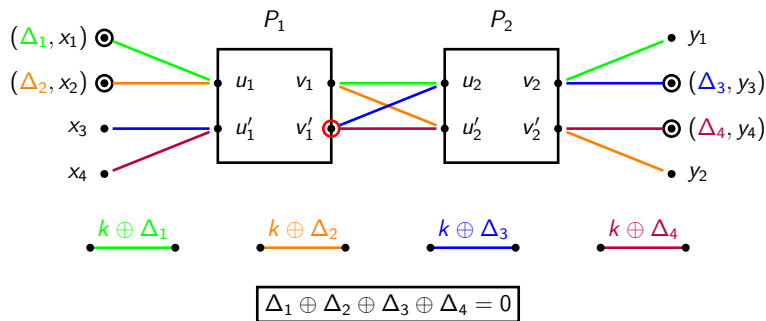
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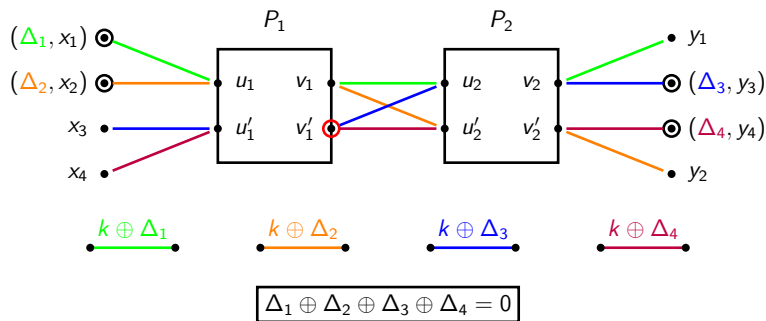
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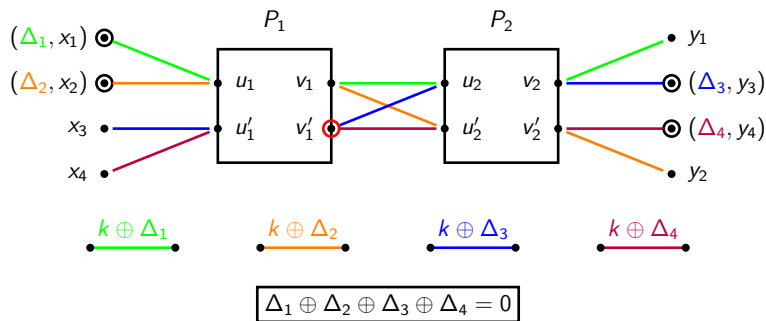
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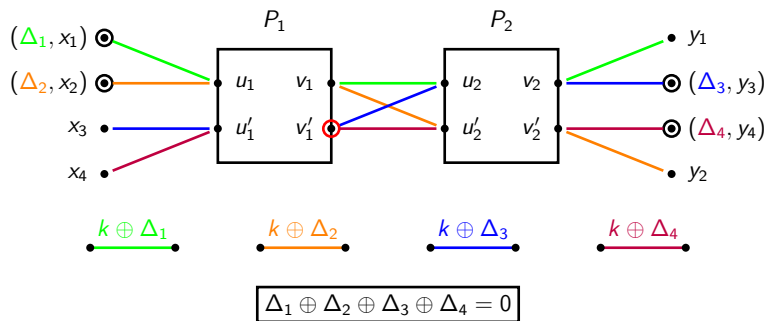
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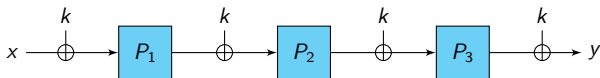
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Security for Three Rounds, Trivial Key-Schedule



Theorem (Cogliati-Seurin [CS15])

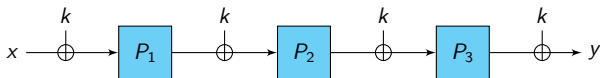
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Proof sketch:

- \mathcal{D} can create forward collisions at P_1 or backward collisions at P_3
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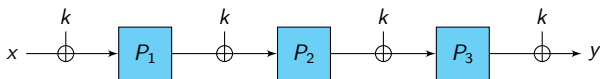
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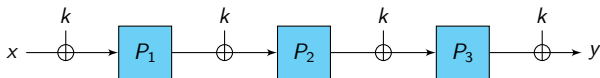
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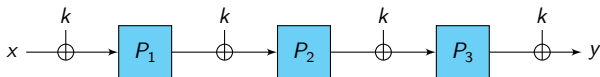
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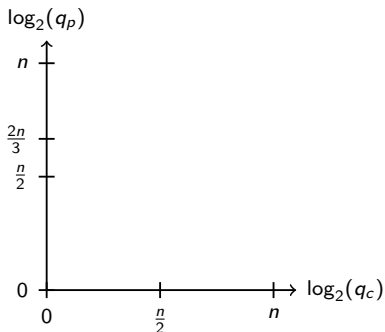
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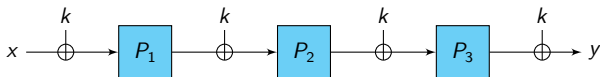
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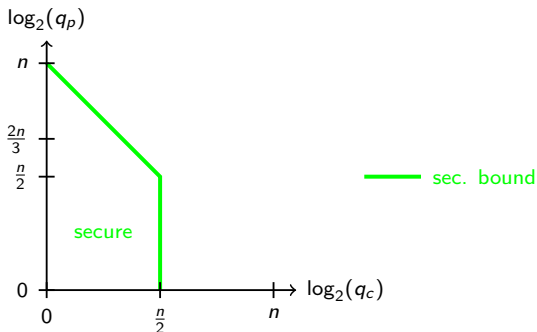
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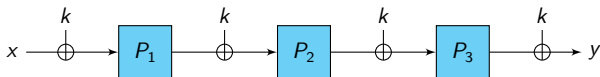
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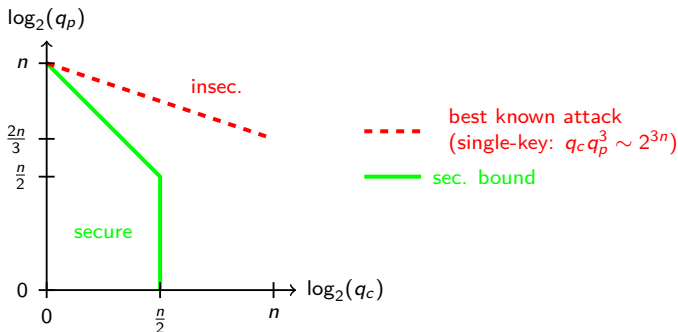
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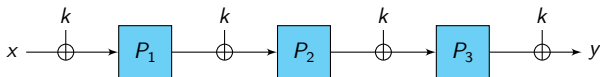
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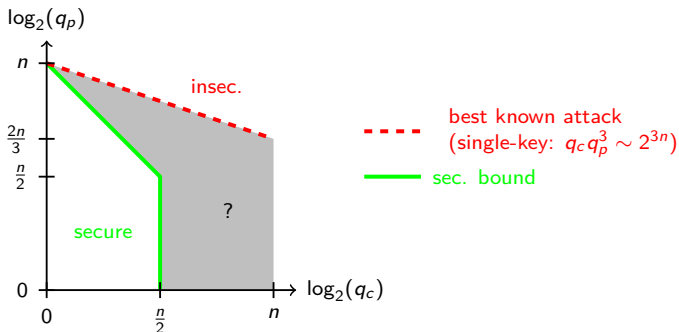
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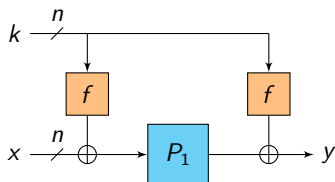
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Security for One Round and a Nonlinear Key-Schedule



Theorem (Cogliati-Seurin [CS15])

For the 1-round EM cipher with key-schedule $f = (f_0, f_1)$:

$$\text{Adv}_{\text{EM}[n,1,f]}^{\text{xor-rka}}(q_c, q_p) \leq \frac{2q_c q_p}{2^n} + \frac{\delta(f)q_c^2}{2^n},$$

where $\delta(f) = \max_{a,b \in \{0,1\}^n, a \neq 0} |\{x \in \{0,1\}^n : f(x \oplus a) \oplus f(x) = b\}|$.

($\delta(f) = 2$ for an APN permutation.)

Some Observations

Application to tweakable block ciphers:

- from any XOR-RKA secure block cipher E , one can construct a tweakable block cipher [LRW02, BK03]

$$\tilde{E}(k, t, x) \stackrel{\text{def}}{=} E(k \oplus t, x)$$



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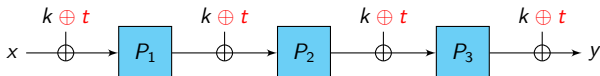
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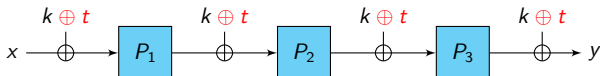
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Security Against Chosen-Key Attacks

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- simply because, e.g., $E_0(0)$ has a specific, non-random value. . .
- OK this does not count
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- OK this does not count
- but what counts as a chosen-key attack exactly?
- rigorous definition possible for a **family of block ciphers** based on some underlying **ideal primitive**
- e.g., **IEM cipher** based on a tuple of **random permutations!**

Formalizing Chosen-Key Attacks

Definition (Evasive relation)

An m -ary relation \mathcal{R} is **(q, ε) -evasive** (w.r.t. an ideal cipher E) if any adversary \mathcal{A} making at most q queries to E finds triples $(k_1, x_1, y_1), \dots, (k_m, x_m, y_m)$ (with $E_{k_i}(x_i) = y_i$) satisfying \mathcal{R} with probability at most ε .

Example

- consider E in Davies-Meyer mode $f(k, x) := E_k(x) \oplus x$
- finding a preimage of 0 for f is a unary $(q, \mathcal{O}(\frac{q}{2^n}))$ -evasive relation for E [BRS02]
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Definition (Correlation Intractability)

A block cipher construction \mathcal{C}^F based on some underlying primitive F is said to be (q, ε) -**correlation intractable** w.r.t. an m -ary relation \mathcal{R} if any adversary \mathcal{A} making at most q queries to F finds triples $(k_1, x_1, y_1), \dots, (k_m, x_m, y_m)$ (with $\mathcal{C}_{k_i}^F(x_i) = y_i$) satisfying \mathcal{R} with probability at most ε .

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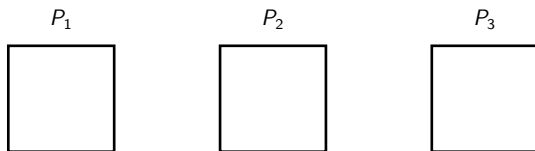
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A Chosen-Key Attack for Three Rounds [LS13]

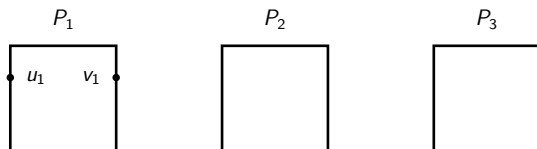


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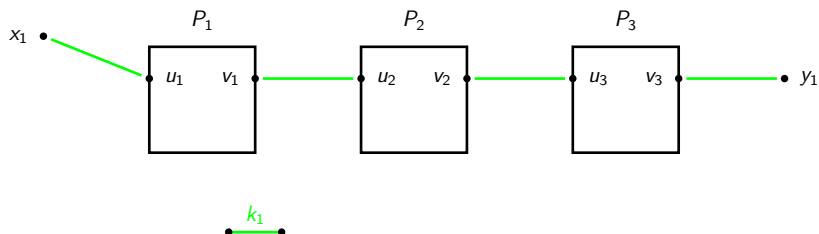


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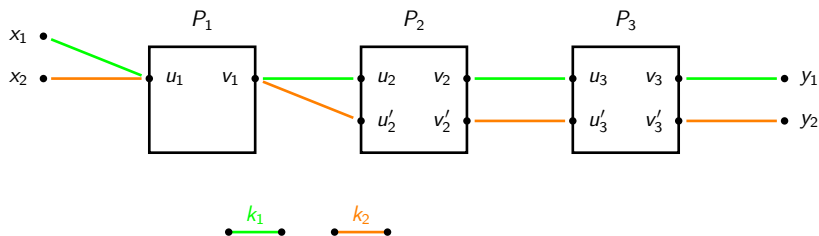


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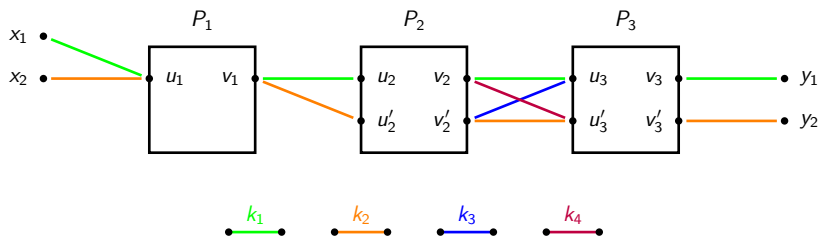


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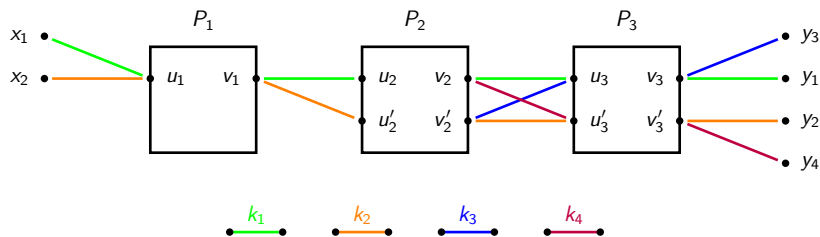


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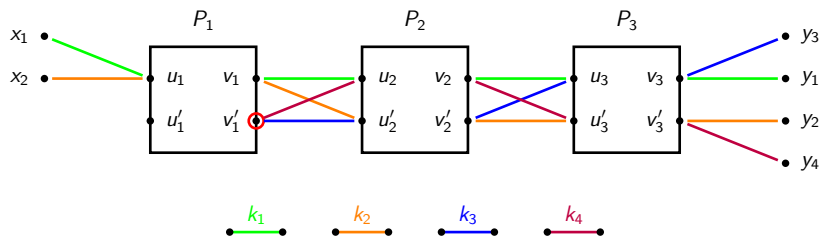


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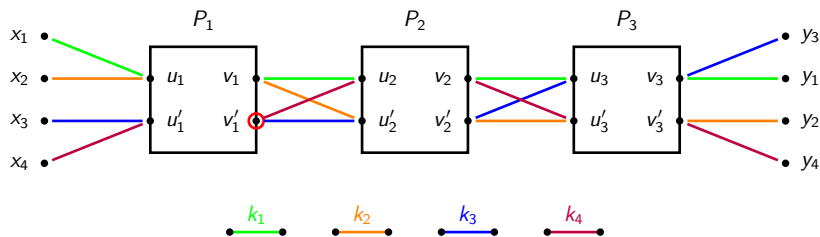


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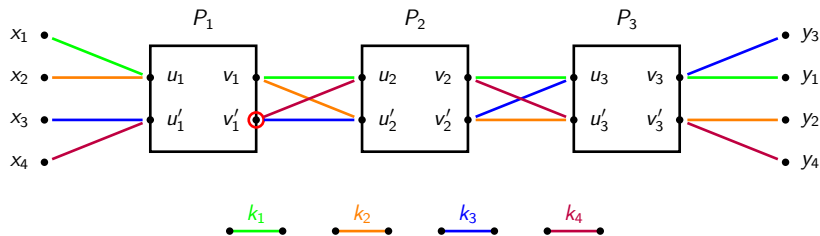


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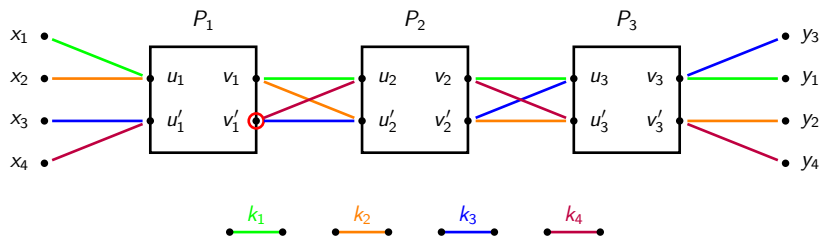


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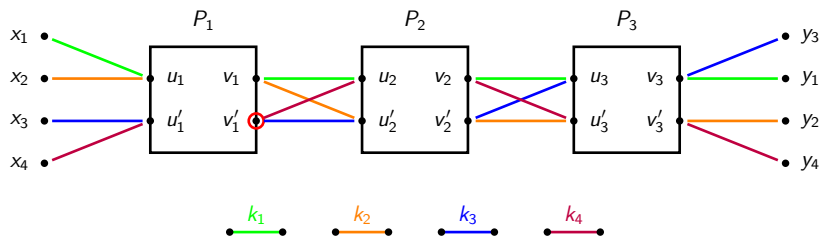


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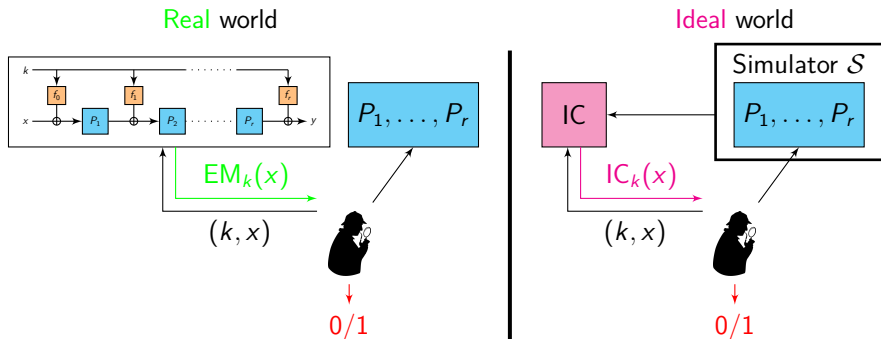


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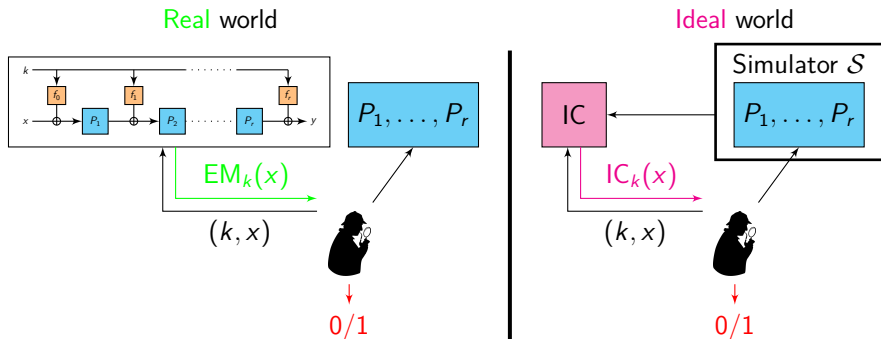
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Proving CKA Resistance: Indifferentiability



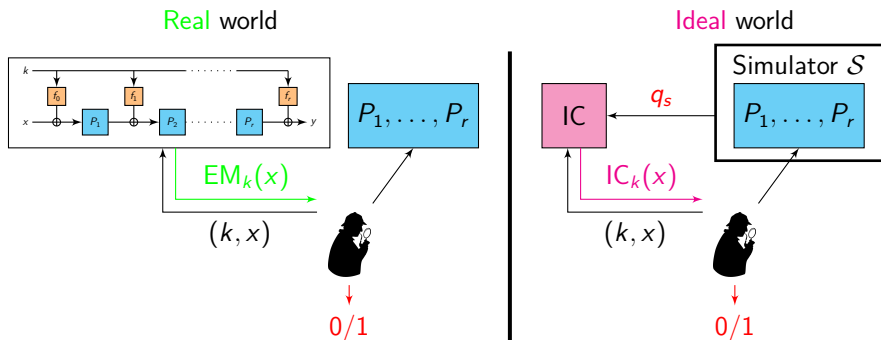
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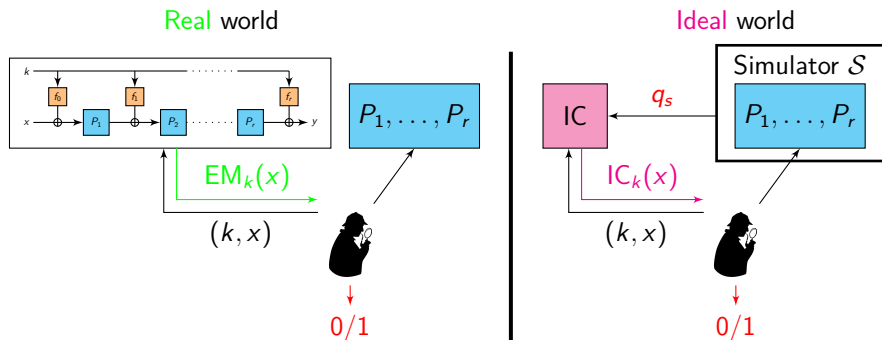
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Definition (Indifferentiability [MRH04])

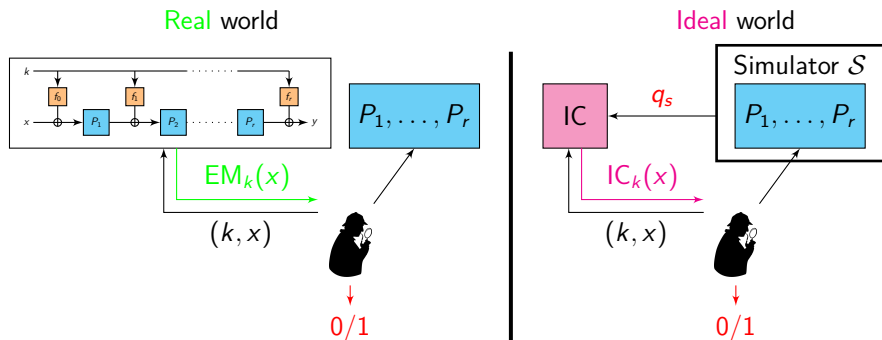
A block cipher construction is said (q_d, q_s, ϵ) -indifferentiable from an ideal cipher if there exists a simulator \mathcal{S} such that for any distinguisher \mathcal{D} making at most q_d queries in total, \mathcal{S} makes at most q_s ideal cipher queries and \mathcal{D} distinguishes the two worlds with adv. at most ϵ .

Two Flavors of Indifferentiability



- **full** indifferentiability: \mathcal{D} can query its oracle as it wishes
- **sequential** indifferentiability: two query phases
 1. \mathcal{D} first queries only P_i 's/ \mathcal{S}
 2. and then only EM/IC
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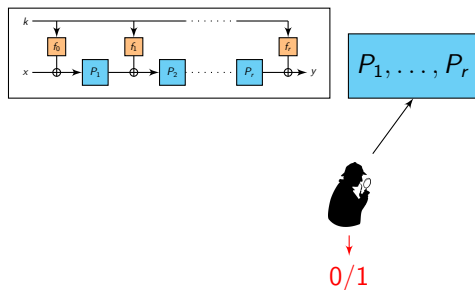
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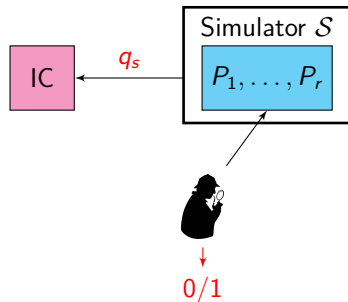
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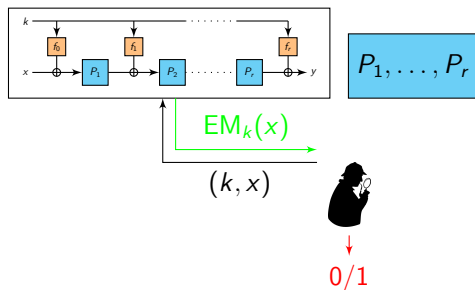
Ideal world



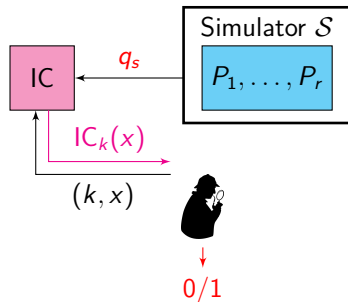
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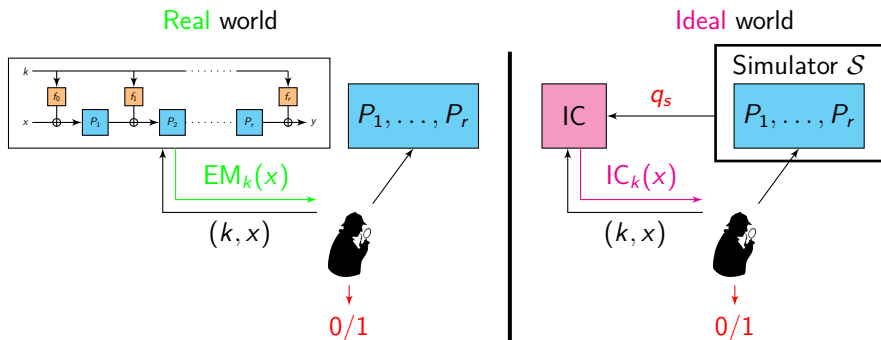


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Theorem (Composition for full indiff. [MRH04])

Informally, if a block cipher construction \mathcal{C}^F is full-indifferentiable from an ideal cipher, then any cryptosystem proven secure with an ideal cipher remains provably secure when used with \mathcal{C}^F (for cryptosystems whose security is defined by a single-stage game [RSS11]).

Theorem (Composition for seq. indiff. [MPS12, CS15])

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queries	q_s	$\xrightarrow{(q_d, q_s, \varepsilon)\text{-seq-indiff.}}$	q_d
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queries	q_s	$\xrightarrow{(q_d, q_s, \varepsilon)\text{-seq-indiff.}}$	q_d
success proba.	ε_{ic}		$\varepsilon_{ic} + \varepsilon$

Indifferentiability Results for the IEM Cipher

Theorem (Andreeva *et al.* [ABD⁺13])

The *5-round IEM cipher* with a key-schedule *modeled as a random oracle* is *fully* indifferentiable from an ideal cipher.

NB: strong assumption on the key-schedule (often invertible in real BCs)

Theorem (Lampe-Seurin [LS13])

The *12-round IEM cipher* with the *trivial* key-schedule is *fully* indifferentiable from an ideal cipher.

Theorem (Cogliati-Seurin [CS15])

The *4-round IEM cipher* with the *trivial* key-schedule is *sequentially* indifferentiable from an ideal cipher with $q_s = \mathcal{O}(q_d^2)$ and $\varepsilon = \mathcal{O}(q_d^4/2^n)$

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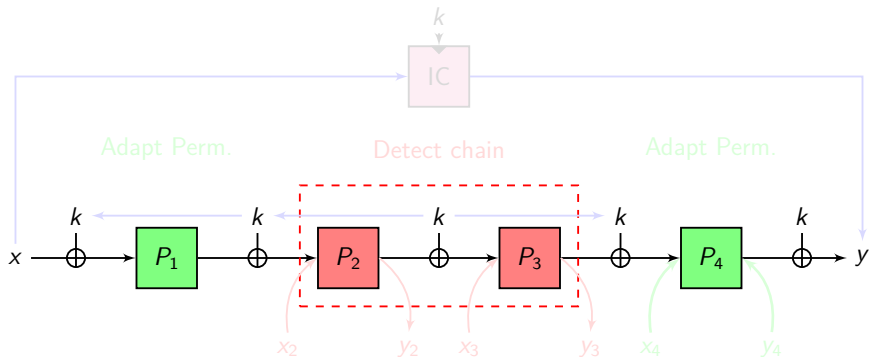
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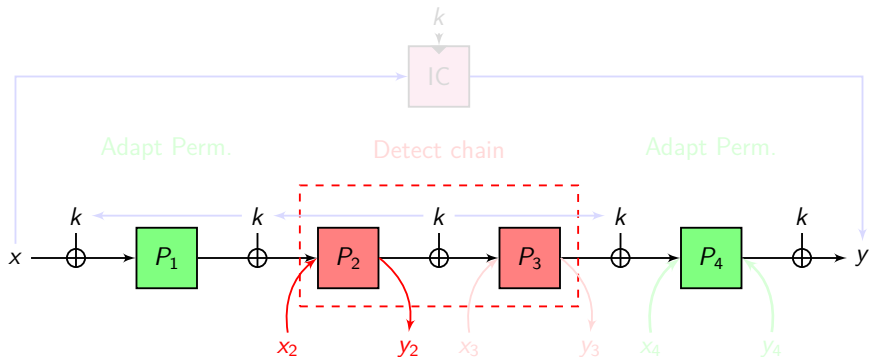
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Seq-indifferentiability for 4 Rounds: Simulator



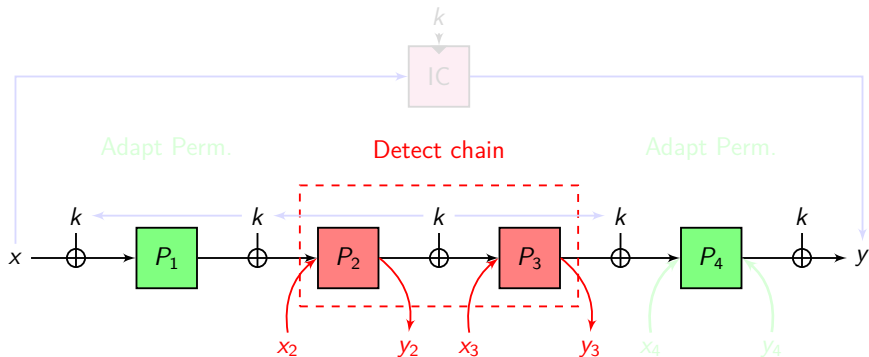
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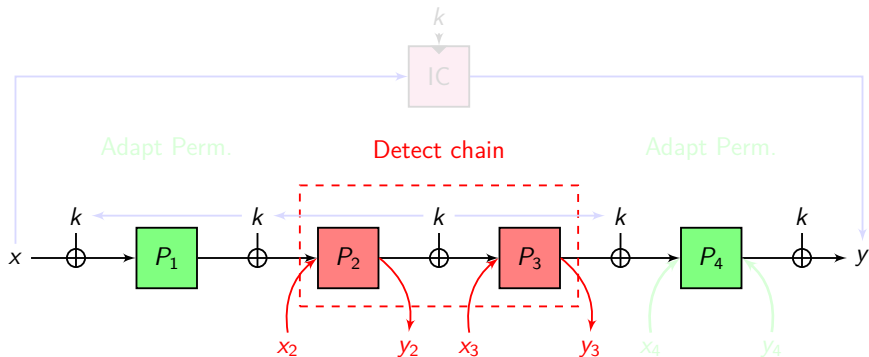
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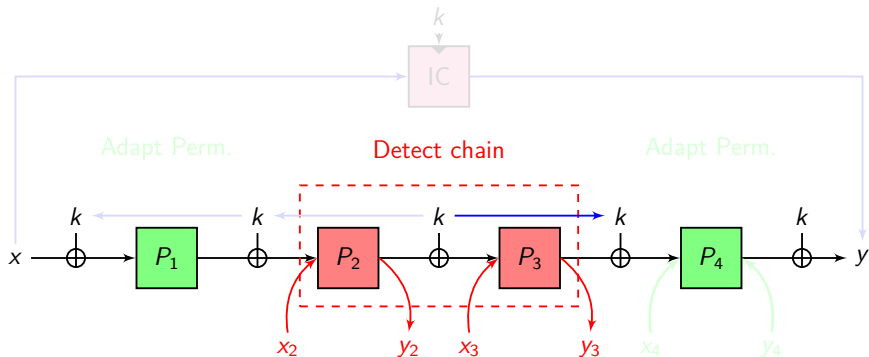
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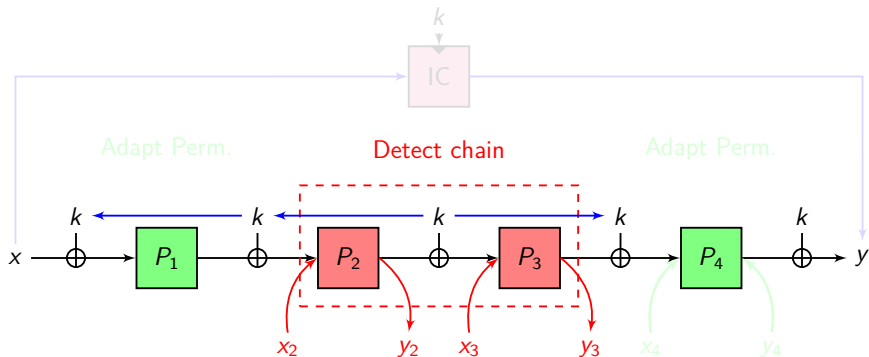
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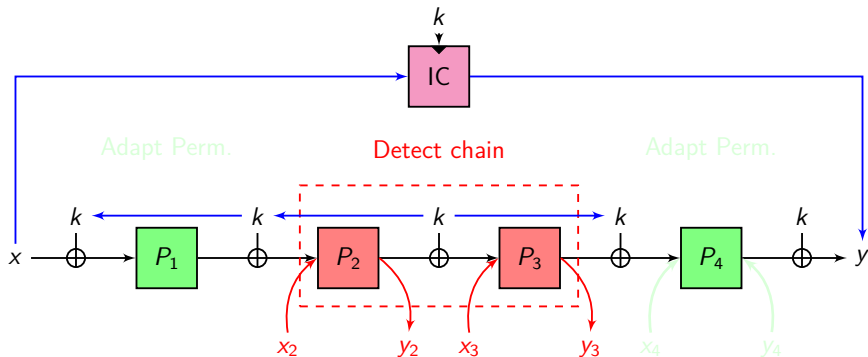
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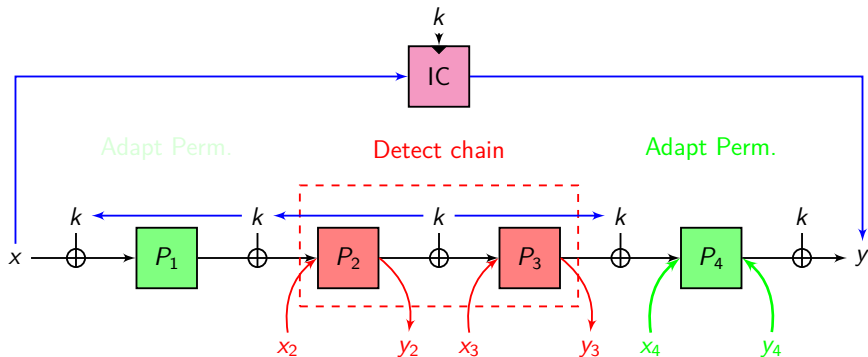
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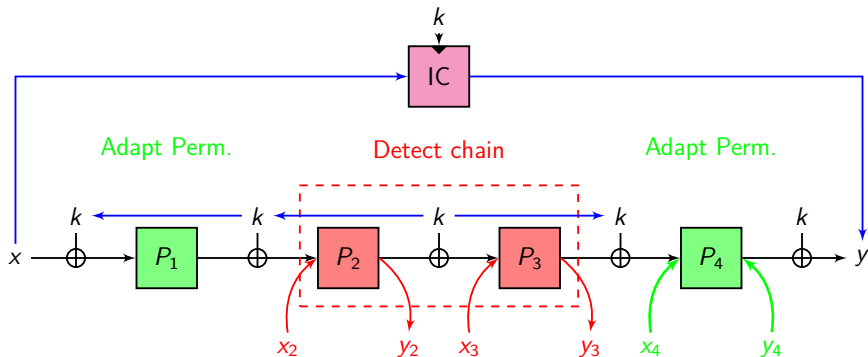
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By the composition theorem “seq-indiff. \Rightarrow correlation-intractability”:

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Let \mathcal{R} be a (q^2, ε_{ic}) -evasive relation w.r.t. an ideal cipher. Then the 4-round IEM with the trivial key-schedule is $(q, \varepsilon_{ic} + \mathcal{O}(\frac{q^4}{2^n}))$ correlation intractable w.r.t. \mathcal{R} .

Example

Consider $f = 4$ -round IEM cipher in Davies-Meyer mode. Then

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Conclusion

Morality:

- **idealized models** can be fruitful
- practical meaning of the results is **debatable**:
 - the high-level structure of SPNs is sound (and may even yield something close to an ideal cipher)
 - says little about concrete block ciphers (inner permutations of, say, AES are too simple)

Open problems:

- RKA security beyond the birthday bound (4 rounds $\rightarrow 2^{\frac{2n}{3}}$ -security?)
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Summary of Known Results

Security notion	# of rounds	Key schedule	Security bound	Simul. (q_S/t_S)	Ref.
Single-key	$r \geq 1$	independent	$2^{\frac{m}{r+1}}$	—	[CS14]
	1	trivial	$2^{\frac{n}{2}}$	—	[EM97, DKS12]
	2	trivial	$2^{\frac{2n}{3}}$	—	[CLL ⁺ 14]
XOR RKA	3	trivial	$2^{\frac{n}{2}}$	—	[CS15, FP15]
	1	nonlinear	$2^{\frac{n}{2}}$	—	[CS15]
CKA (Seq-ind.)	4	trivial	$2^{\frac{n}{4}}$	q^2 / q^2	[CS15]
Full indiff.	5	rand. oracle	$2^{\frac{n}{10}}$	q^2 / q^3	[ABD ⁺ 13]
	12	trivial	$2^{\frac{n}{12}}$	q^4 / q^6	[LS13]

The End...

Thanks for your attention!

Comments or questions?

References I



Elena Andreeva, Andrey Bogdanov, Yevgeniy Dodis, Bart Mennink, and John P. Steinberger. On the Indifferentiability of Key-Alternating Ciphers. In Ran Canetti and Juan A. Garay, editors, *Advances in Cryptology - CRYPTO 2013 (Proceedings, Part I)*, volume 8042 of *LNCS*, pages 531–550. Springer, 2013. Full version available at <http://eprint.iacr.org/2013/061>.



Mihir Bellare and Tadayoshi Kohno. A Theoretical Treatment of Related-Key Attacks: RKA-PRPs, RKA-PRFs, and Applications. In Eli Biham, editor, *Advances in Cryptology - EUROCRYPT 2003*, volume 2656 of *LNCS*, pages 491–506. Springer, 2003.



John Black, Phillip Rogaway, and Thomas Shrimpton. Black-Box Analysis of the Block-Cipher-Based Hash-Function Constructions from PGV. In Moti Yung, editor, *Advances in Cryptology - CRYPTO 2002*, volume 2442 of *LNCS*, pages 320–335. Springer, 2002.

References II

-  Shan Chen, Rodolphe Lampe, Jooyoung Lee, Yannick Seurin, and John P. Steinberger. Minimizing the Two-Round Even-Mansour Cipher. In Juan A. Garay and Rosario Gennaro, editors, *Advances in Cryptology - CRYPTO 2014 (Proceedings, Part I)*, volume 8616 of *LNCS*, pages 39–56. Springer, 2014. Full version available at <http://eprint.iacr.org/2014/443>.
-  Shan Chen and John Steinberger. Tight Security Bounds for Key-Alternating Ciphers. In Phong Q. Nguyen and Elisabeth Oswald, editors, *Advances in Cryptology - EUROCRYPT 2014*, volume 8441 of *LNCS*, pages 327–350. Springer, 2014. Full version available at <http://eprint.iacr.org/2013/222>.
-  Benoît Cogliati and Yannick Seurin. On the Provable Security of the Iterated Even-Mansour Cipher against Related-Key and Chosen-Key Attacks. In *EUROCRYPT 2015*, 2015. To appear. Full version available at <http://eprint.iacr.org/2015/069>.
-  Orr Dunkelman, Nathan Keller, and Adi Shamir. Minimalism in Cryptography: The Even-Mansour Scheme Revisited. In David Pointcheval and Thomas Johansson, editors, *Advances in Cryptology - EUROCRYPT 2012*, volume 7237 of *LNCS*, pages 336–354. Springer, 2012.

References III

-  Shimon Even and Yishay Mansour. A Construction of a Cipher from a Single Pseudorandom Permutation. *Journal of Cryptology*, 10(3):151–162, 1997.
-  Pooya Farshim and Gordon Procter. The Related-Key Security of Iterated Even-Mansour Ciphers. In *Fast Software Encryption - FSE 2015*, 2015. To appear. Full version available at <http://eprint.iacr.org/2014/953>.
-  Joe Kilian and Phillip Rogaway. How to Protect DES Against Exhaustive Key Search (an Analysis of DESX). *Journal of Cryptology*, 14(1):17–35, 2001.
-  Moses Liskov, Ronald L. Rivest, and David Wagner. Tweakable Block Ciphers. In Moti Yung, editor, *Advances in Cryptology - CRYPTO 2002*, volume 2442 of LNCS, pages 31–46. Springer, 2002.
-  Rodolphe Lampe and Yannick Seurin. How to Construct an Ideal Cipher from a Small Set of Public Permutations. In Kazue Sako and Palash Sarkar, editors, *Advances in Cryptology - ASIACRYPT 2013 (Proceedings, Part I)*, volume 8269 of LNCS, pages 444–463. Springer, 2013. Full version available at <http://eprint.iacr.org/2013/255>.

References IV



Avradip Mandal, Jacques Patarin, and Yannick Seurin. On the Public Indifferentiability and Correlation Intractability of the 6-Round Feistel Construction. In Ronald Cramer, editor, *Theory of Cryptography Conference - TCC 2012*, volume 7194 of *LNCS*, pages 285–302. Springer, 2012. Full version available at <http://eprint.iacr.org/2011/496>.



Ueli M. Maurer, Renato Renner, and Clemens Holenstein. Indifferentiability, Impossibility Results on Reductions, and Applications to the Random Oracle Methodology. In Moni Naor, editor, *Theory of Cryptography Conference- TCC 2004*, volume 2951 of *LNCS*, pages 21–39. Springer, 2004.



Thomas Ristenpart, Hovav Shacham, and Thomas Shrimpton. Careful with Composition: Limitations of the Indifferentiability Framework. In Kenneth G. Paterson, editor, *Advances in Cryptology - EUROCRYPT 2011*, volume 6632 of *LNCS*, pages 487–506. Springer, 2011.