Security Analysis of Key-Alternating Feistel Ciphers

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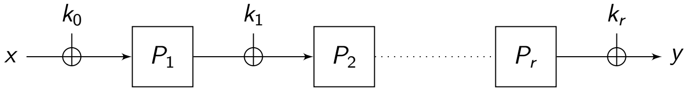
Key-Alternating Ciphers (*aka* iterated Even-Mansour)



- $P_1, \ldots, P_r$ are modeled as public random permutation oracles
- interpretation: gives a guarantee against any adversary which does not use particular properties of the P_i's

Results on the pseudorandomness of KA ciphers

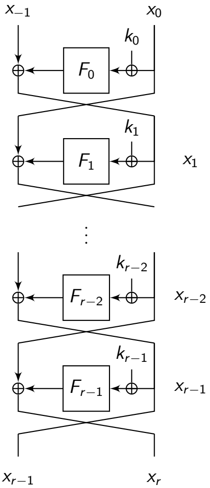
The following results have been successively obtained for the pseudorandomness of KA ciphers (notation: $N = 2^n$):

- for $r = 1$ round, security up to $\mathcal{O}(N^{\frac{1}{2}})$ queries [EM97]
- for $r \geq 2$, security up to $\mathcal{O}(N^{\frac{2}{3}})$ queries [BKL$^+$12]
- for $r \geq 3$, security up to $\mathcal{O}(N^{\frac{3}{4}})$ queries [Ste12]
- for any even r, security up to $\mathcal{O}(N^{\frac{r}{r+2}})$ queries [LPS12]
- tight result: for r rounds, security up to $\mathcal{O}(N^{\frac{r}{r+1}})$ queries [CS13]

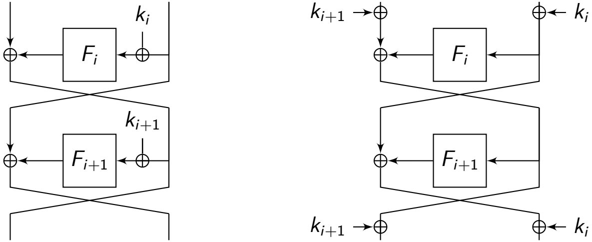
NB: Results for independent round keys $(k_0, k_1, \ldots, k_r)$

Key-Alternating Feistel Ciphers

- functions F_i are public random oracles
- different from the Luby-Rackoff setting (where the F_i's are pseudorandom)



KAF ciphers as a special type of Key-Alternating ciphers



Two rounds of a KAF cipher is equivalent to a 1-round KA cipher where the permutation is a two-round (un-keyed) Feistel cipher with public random functions

Results

- previous results: Gentry and Ramzan [GR04]: secure up to $N^{1/2}$ queries for $r = 4$ rounds
- our results: secure up to $N^{\frac{t}{t+1}}$ queries where

$$t = \left\lfloor \frac{r}{3} \right\rfloor \quad \text{for NCPA attacks}$$

$$t = \left\lfloor \frac{r}{6} \right\rfloor \quad \text{for CCA attacks}$$

- improved results in the Luby-Rackoff setting: security up to $N^{\frac{t}{t+1}}$ queries where

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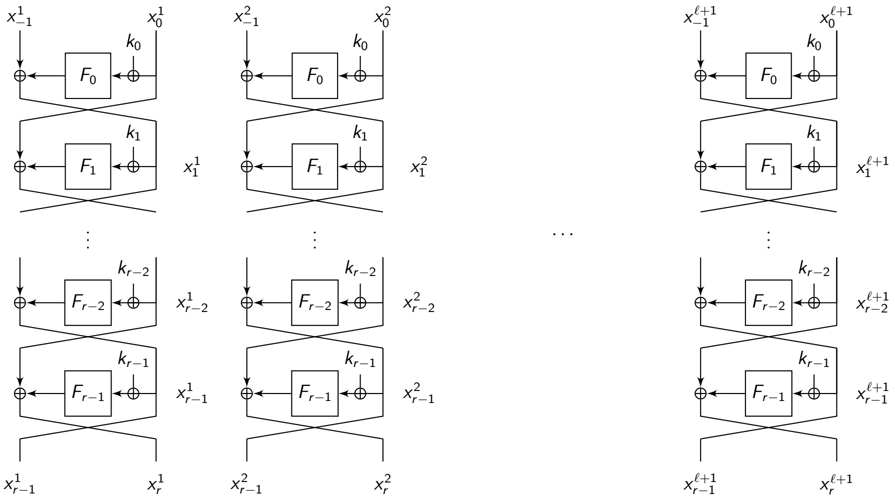

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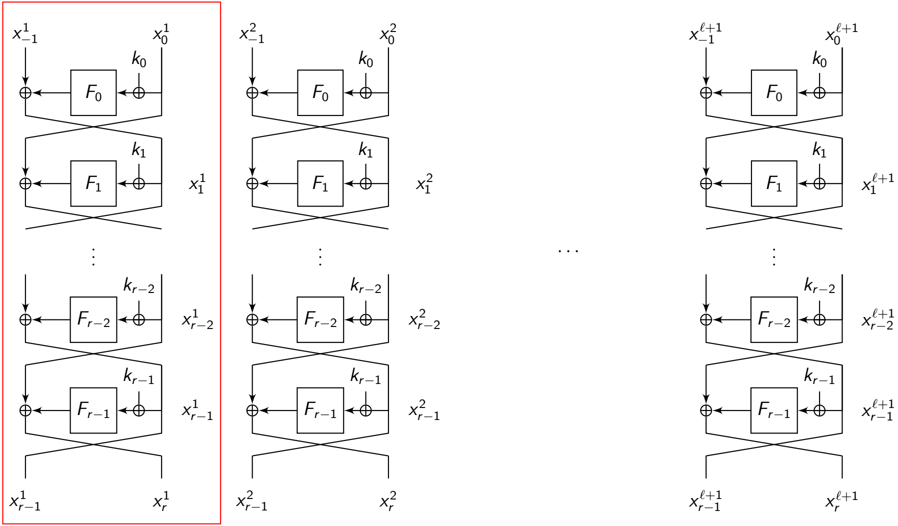

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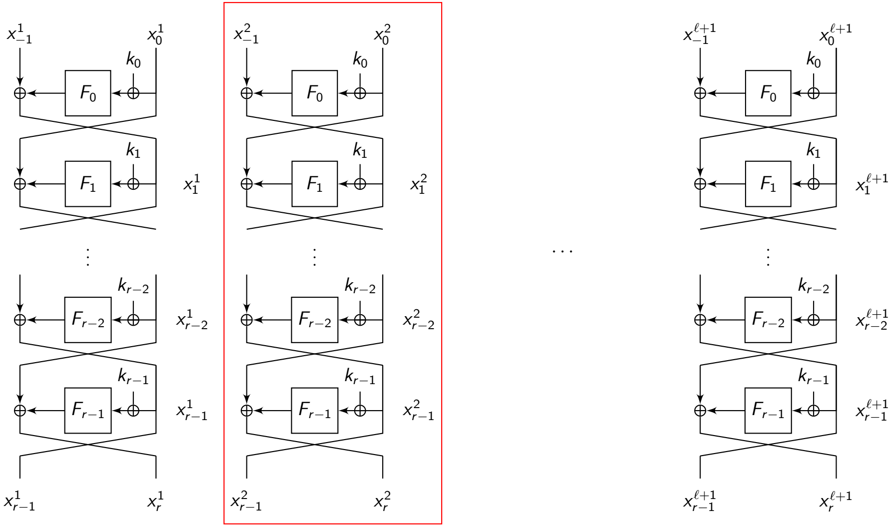

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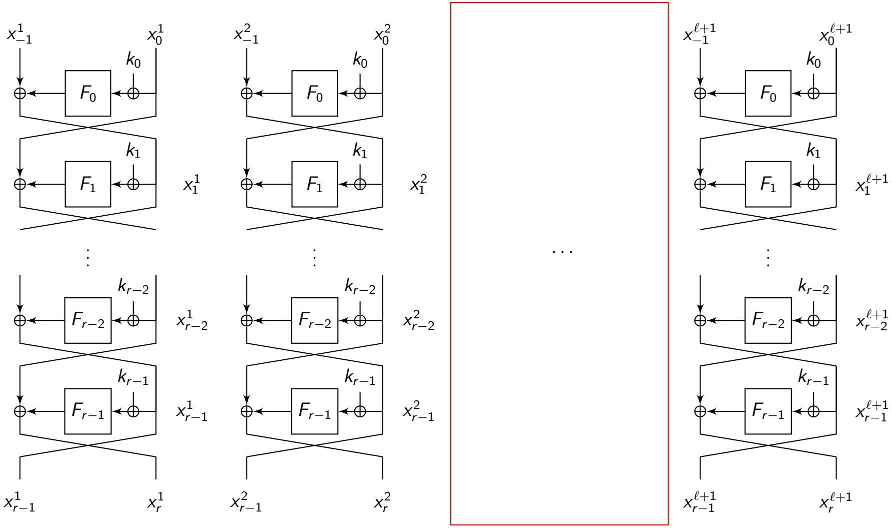

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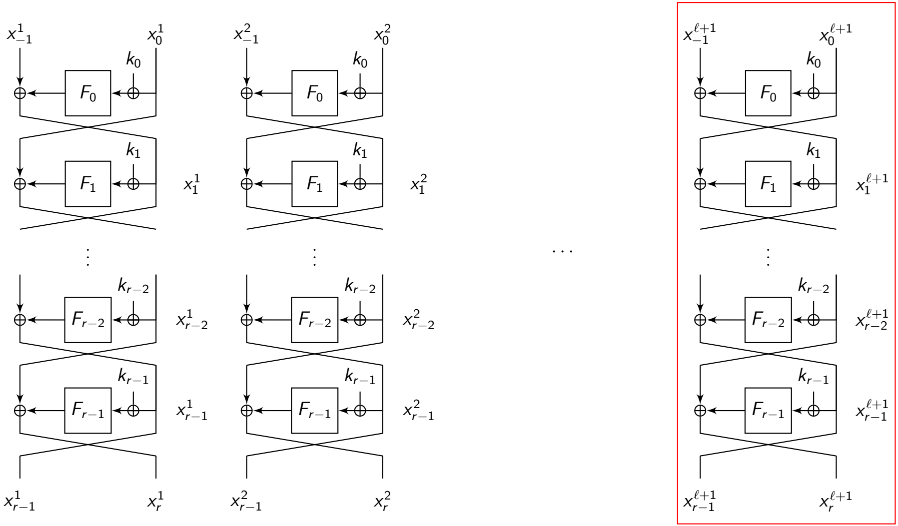

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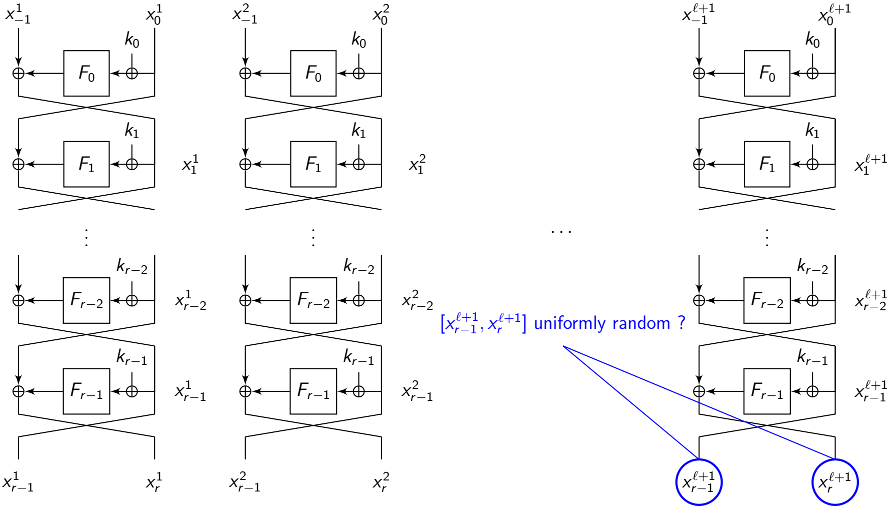

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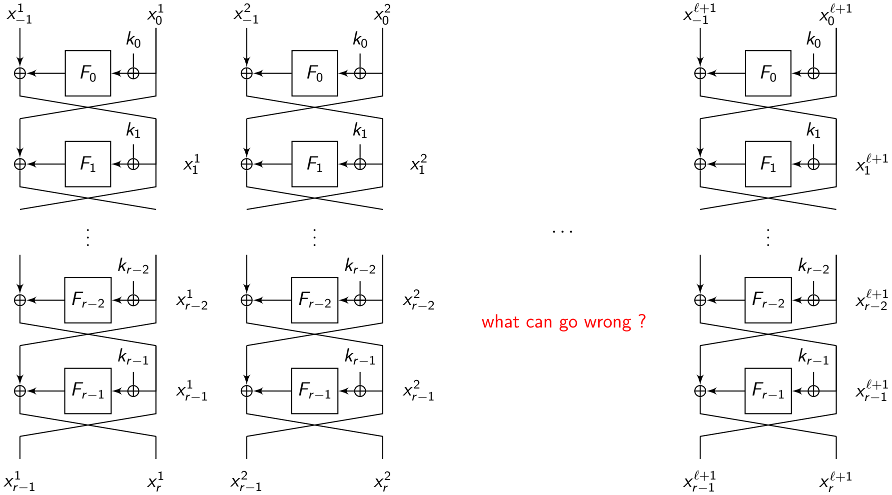

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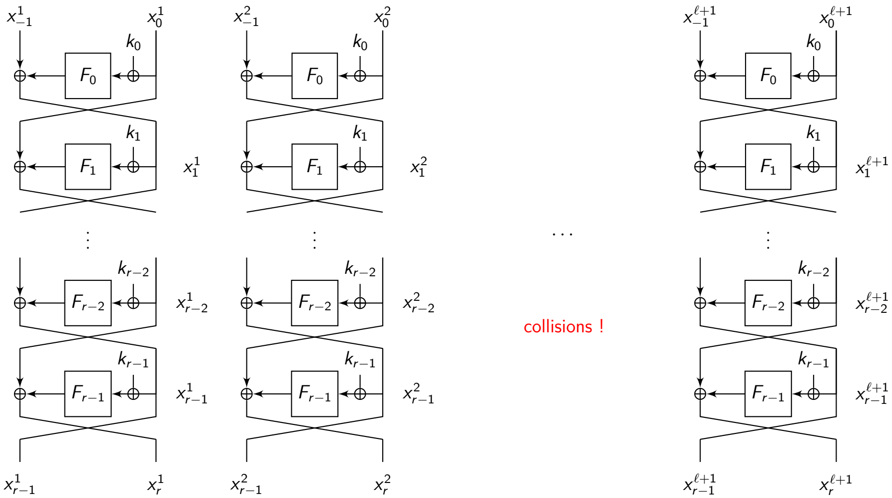

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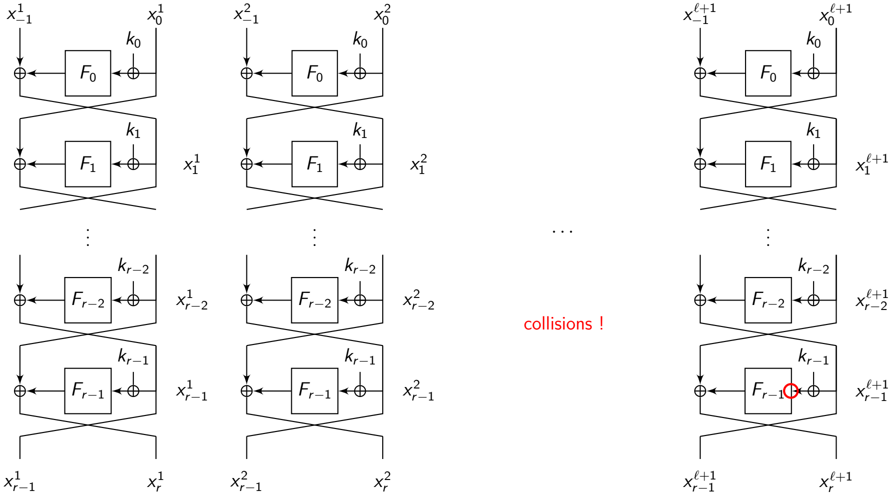

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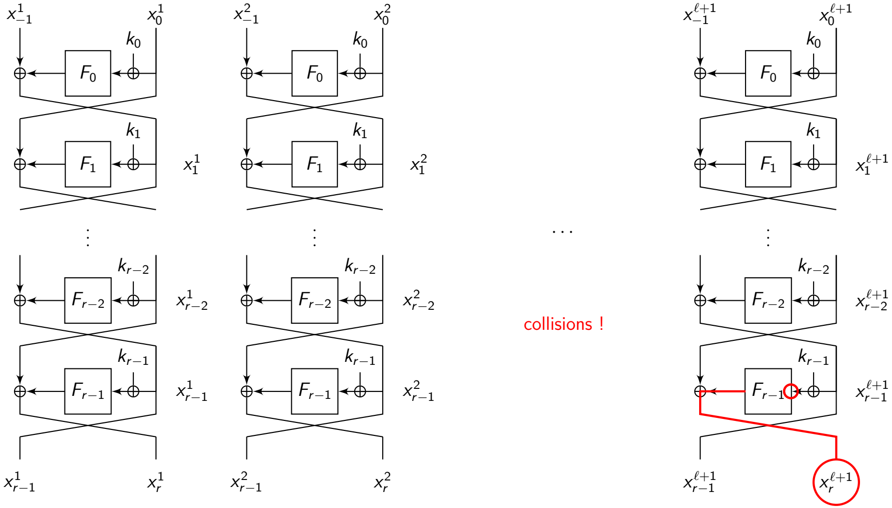

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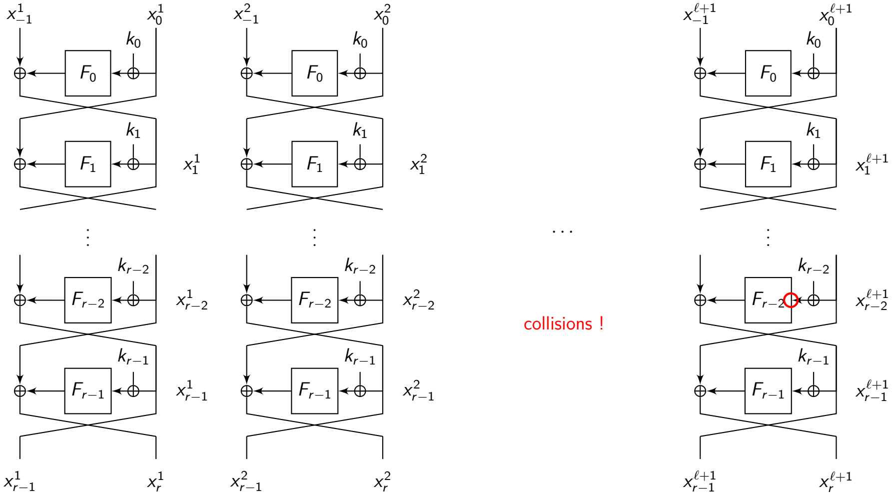

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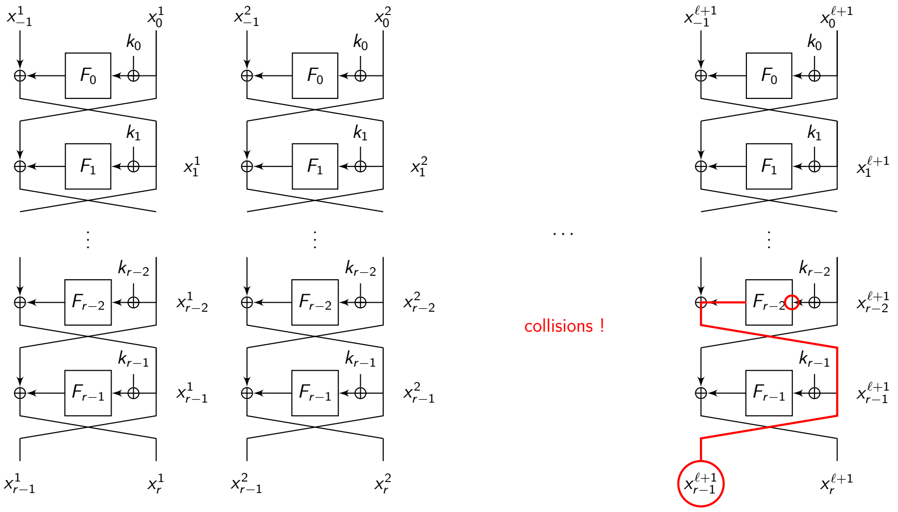

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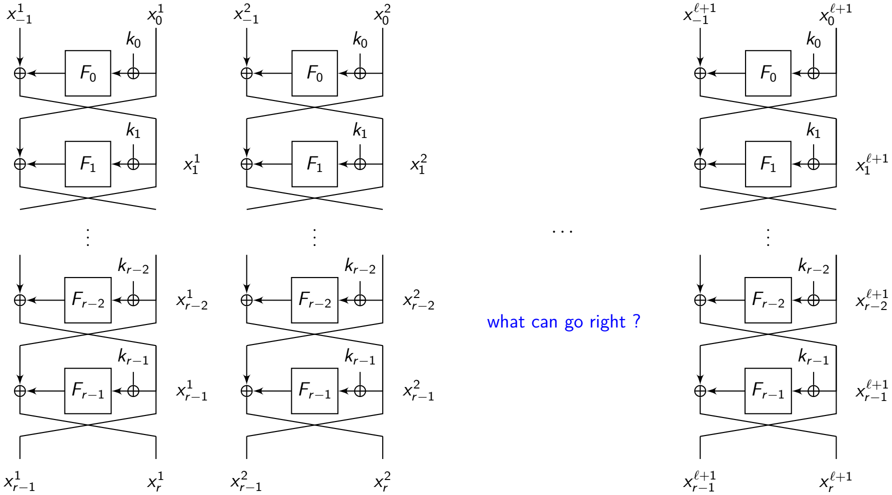

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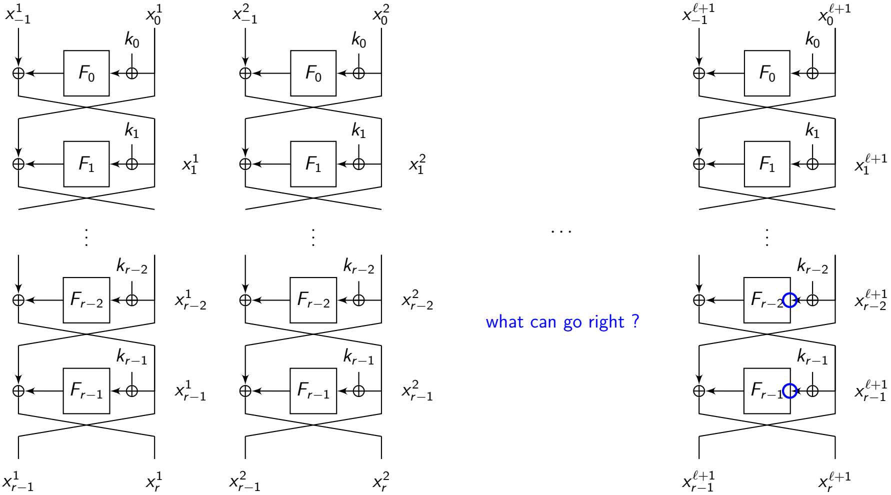

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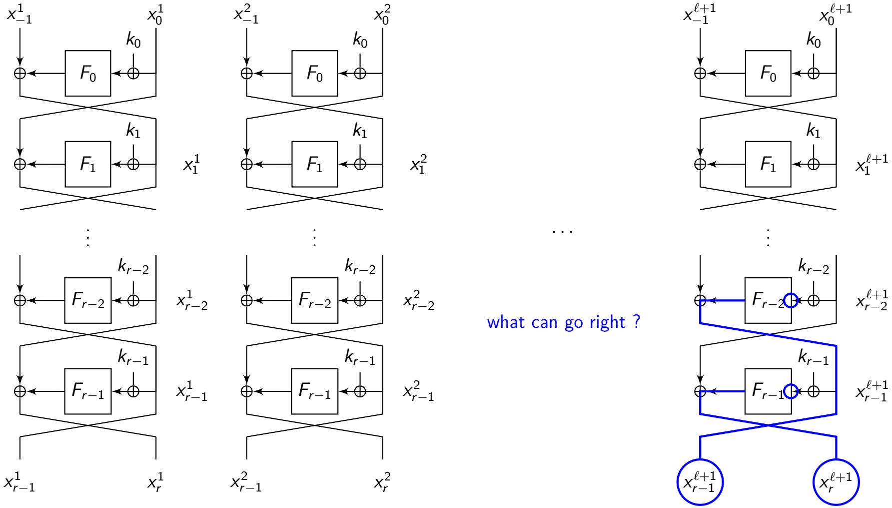

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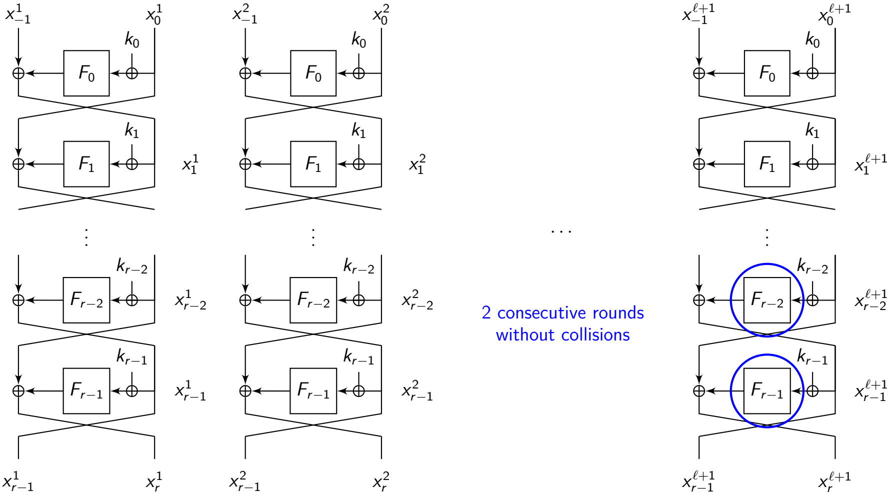

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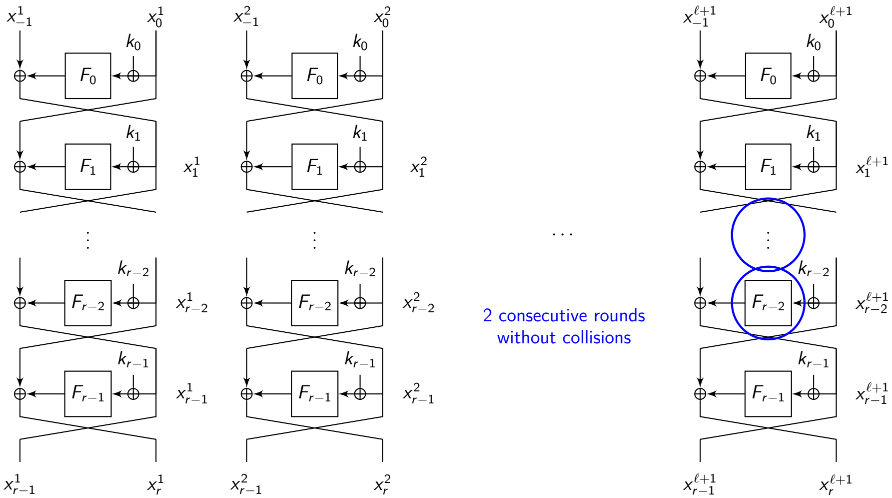

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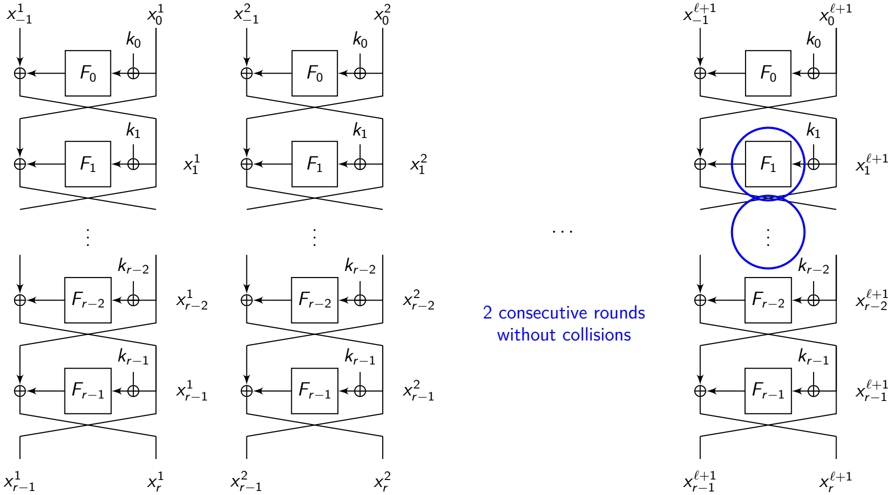

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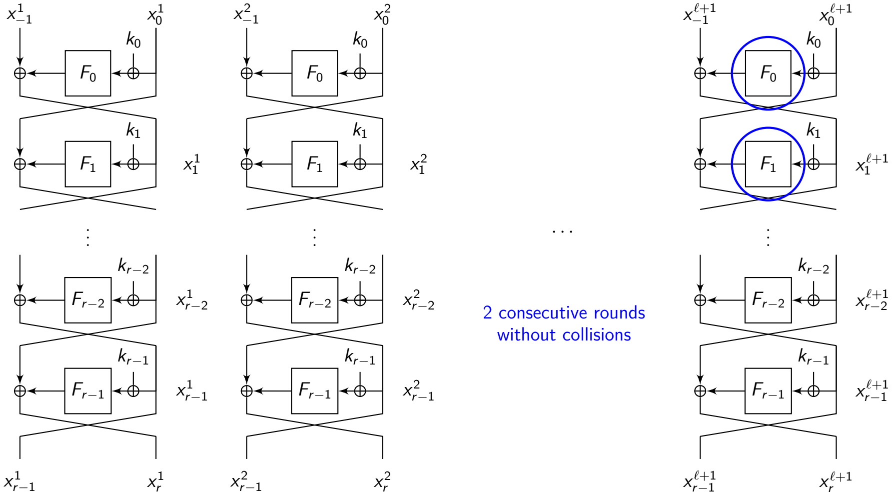

Technique

- Proof using the coupling technique
- main problem: given ℓ queries, upper bound the probability that, for every two consecutive rounds, the $\ell+1$-th query collision in (at least) one of the two rounds.
- A_i = event that the ℓ-th query collisions with previous queries at round i; we want to upper bound

$$\Pr\left[(A_1 \cup A_2) \cap (A_2 \cup A_3) \cap \cdots \cap (A_{r-2} \cup A_{r-1}) \cap (A_{r-1} \cup A_r)\right]$$

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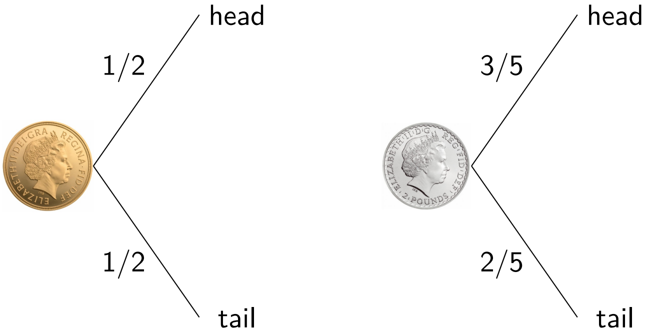

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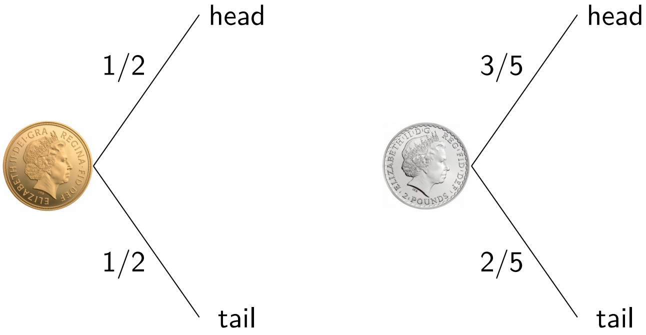

$$\text{Adv} = \text{Statistical distance} = \left|\frac{3}{5} - \frac{1}{2}\right| = \frac{1}{10}$$

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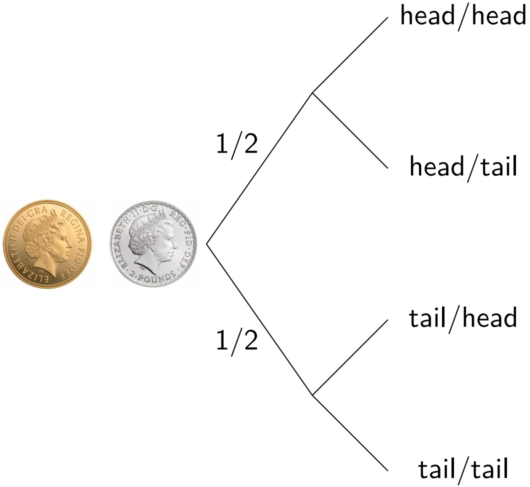

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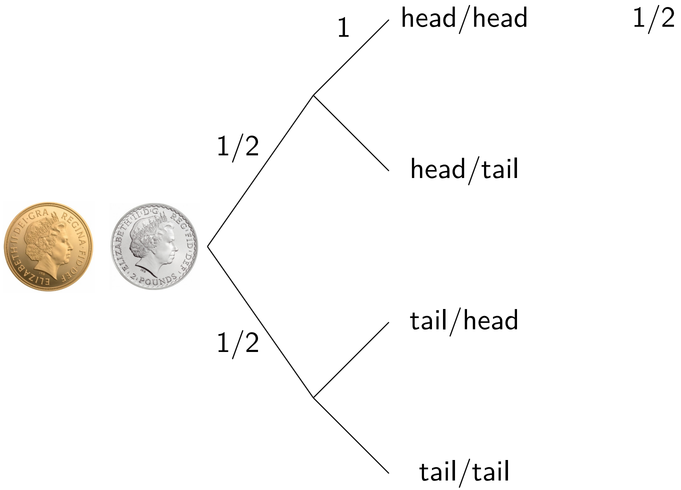

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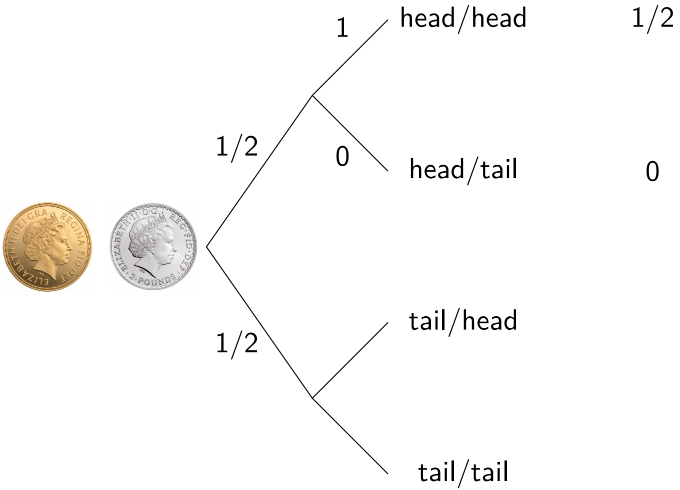

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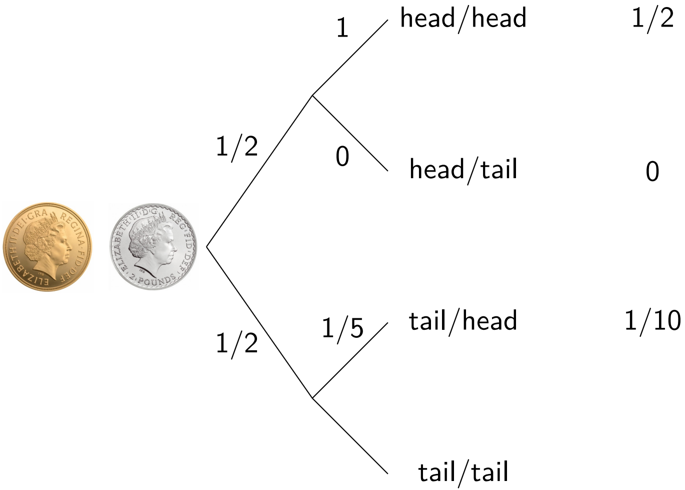

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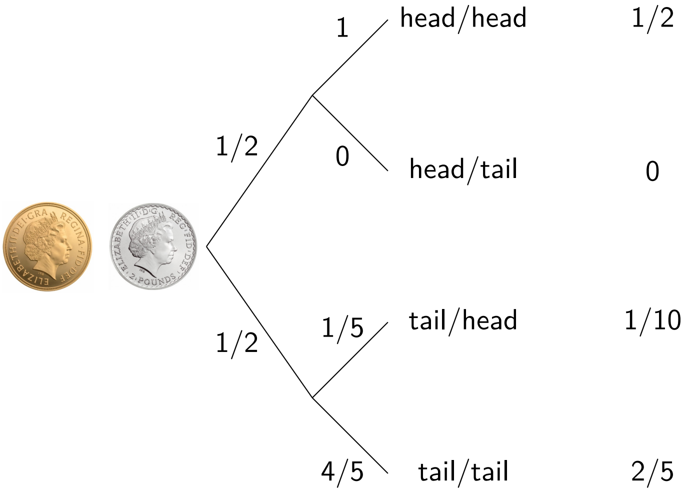

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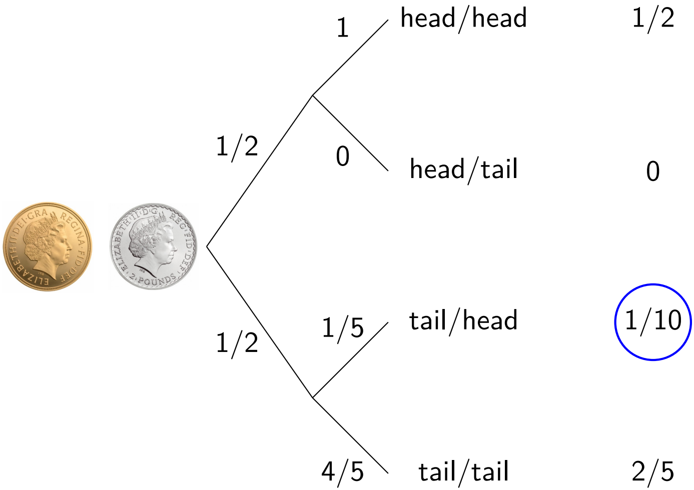

The Coupling technique

random variables	X	Y
probability distributions	μ	ν

The Coupling lemma

$$\|\mu - \nu\| \leq \Pr[X \neq Y]$$

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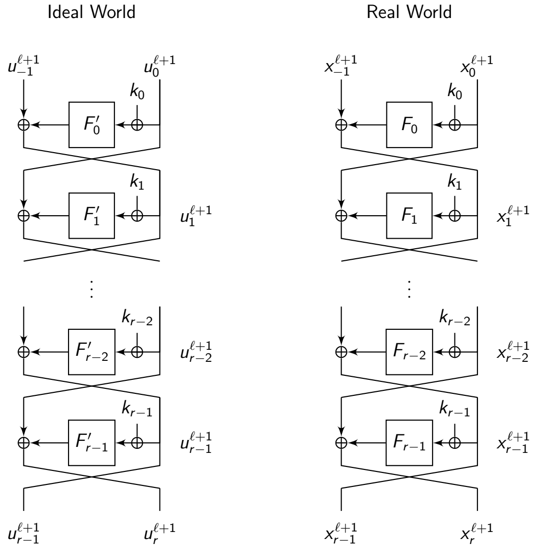
The Coupling Technique for the KAF



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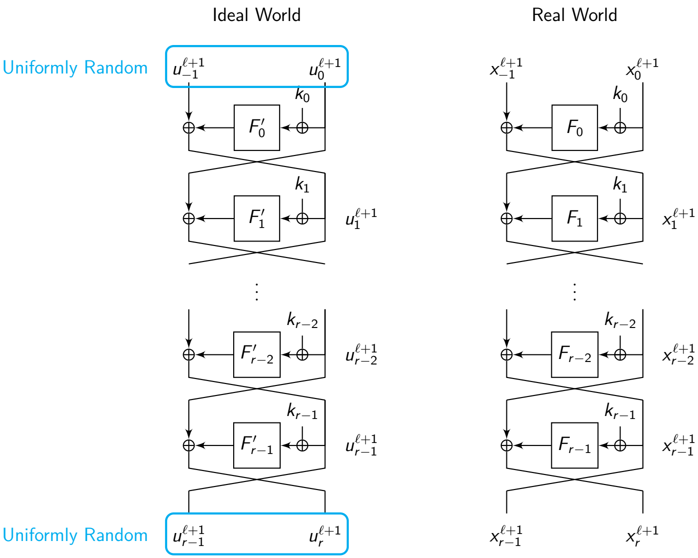

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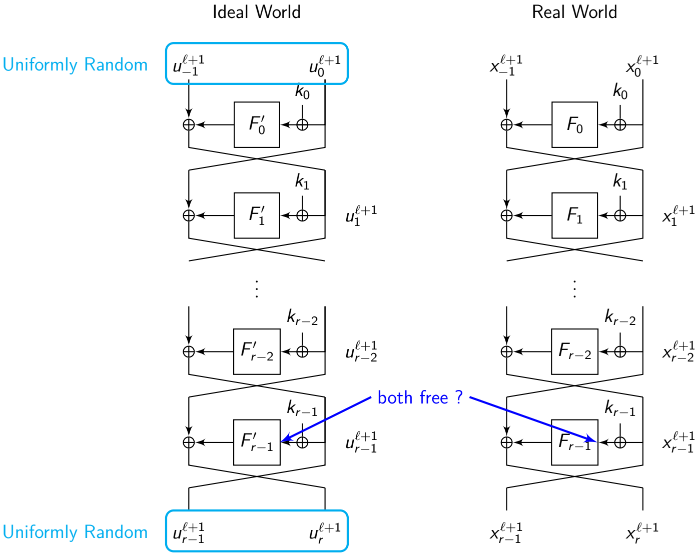

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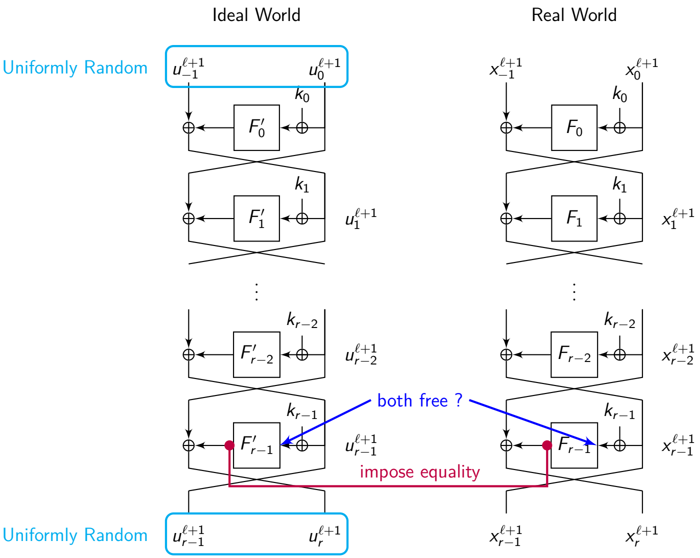

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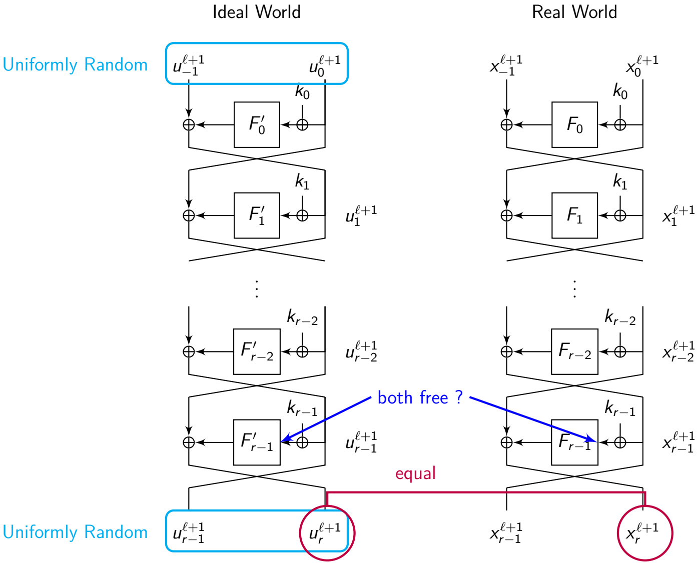

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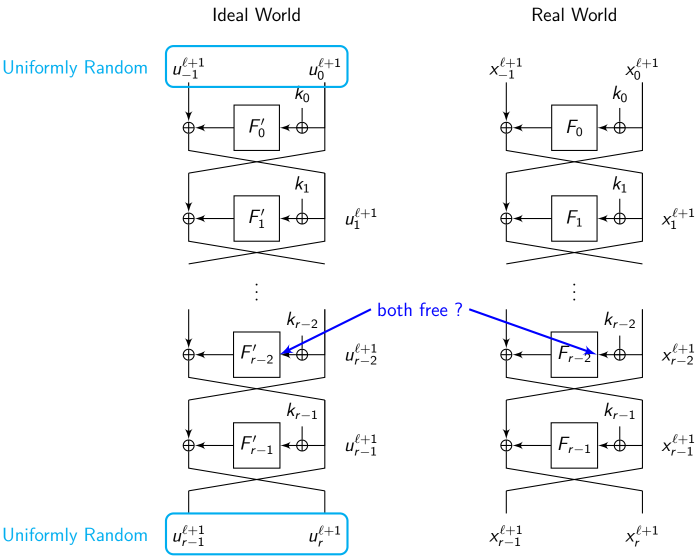

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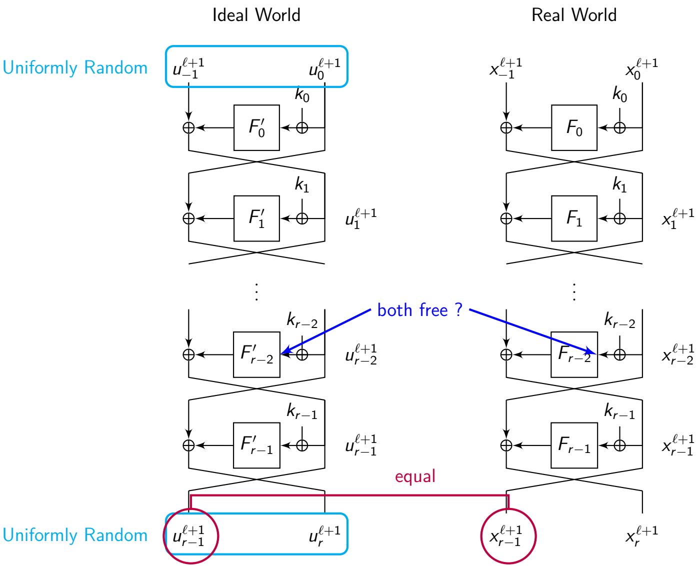

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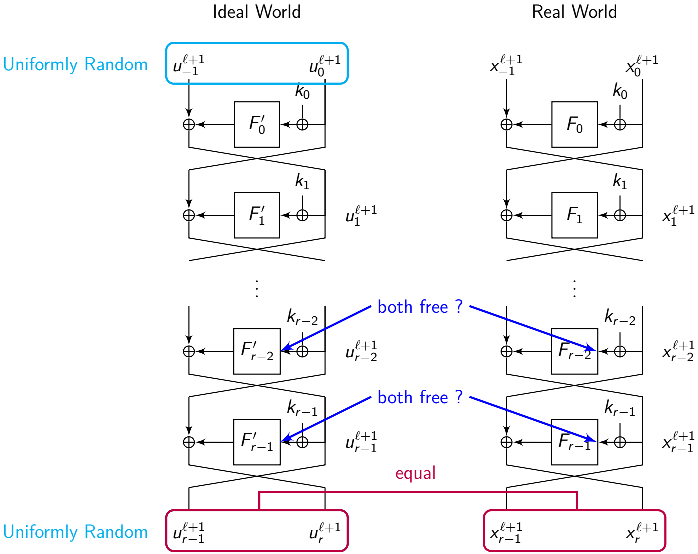

Advantage

q_e :number of queries to the cipher

q_f :number of queries to the round functions

$$\mathbf{Adv}^{\mathrm{ncpa}}_{\mathtt{KAF}[n,r]}(q_e, q_f) \leq \frac{4^t}{t+1} \frac{(q_e + 2q_f)^{t+1}}{2^{tn}} \quad \text{with} \quad t = \left\lfloor \frac{r}{3} \right\rfloor.$$

$$\mathbf{Adv}^{\mathrm{cca}}_{\mathtt{KAF}[n,2r']}(q_e, q_f) \leq 4 \left(\frac{4^t}{t+1} \frac{(q_e + 2q_f)^{t+1}}{2^{tn}} \right)^{1/2} \quad \text{with} \quad t = \left\lfloor \frac{r'}{3} \right\rfloor.$$

The end...

Thanks for your attention!
Comments or questions?

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