Relaxing Full-Codebook Security: A Refined Analysis of Key-Length Extension Schemes

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Outline

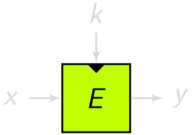
Context: Key-Length Extension for Block Ciphers

Main Lemma

Randomized Cascading

Plain Cascading

Block Ciphers



A block cipher E

- takes as input
  - a plaintext $x \in \{0,1\}^n$
  - a key $k \in \{0,1\}^\kappa$
- outputs a ciphertext $y \in \{0,1\}^n$
- $E_k(\cdot)$ is a permutation $\forall k$
- examples: DES, AES, IDEA, etc.

Notation

- n = block-length
- κ = key-length

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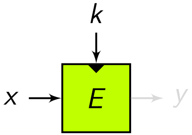

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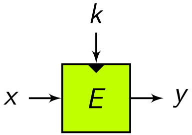

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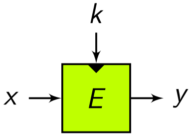

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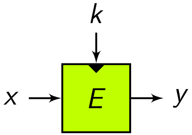

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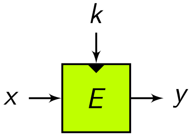

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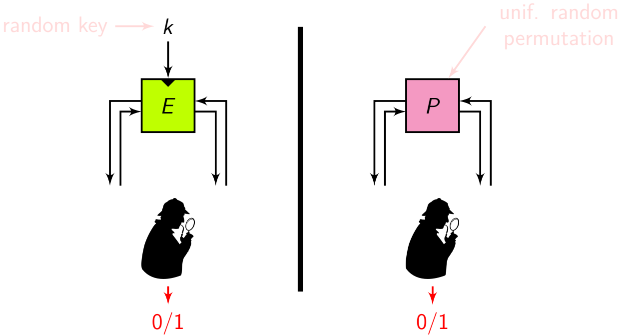
Block Cipher Security: Pseudorandom Permutations



SPRP (*a.k.a.* CCA) advantage:

$$\mathbf{Adv}_{E}^{\mathrm{sprp}}(\mathcal{D}) = \left|\Pr\left[\mathcal{D}^{E_k} = 1\right] - \Pr\left[\mathcal{D}^{P} = 1\right]\right|$$

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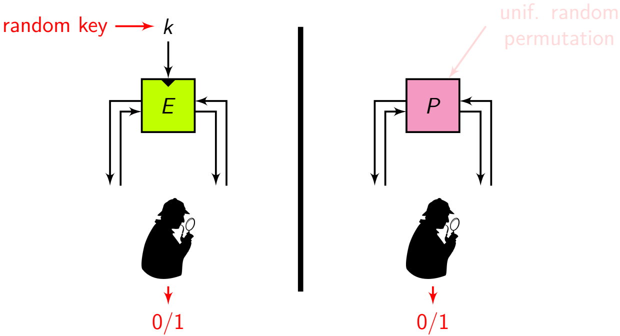

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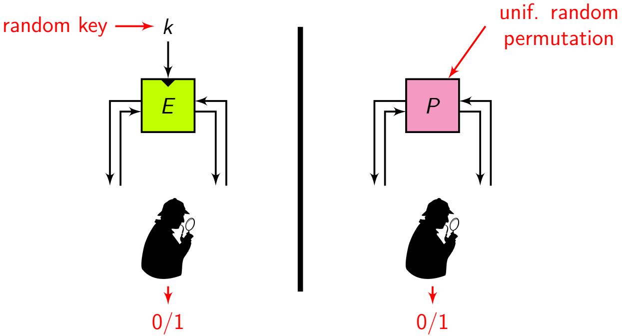

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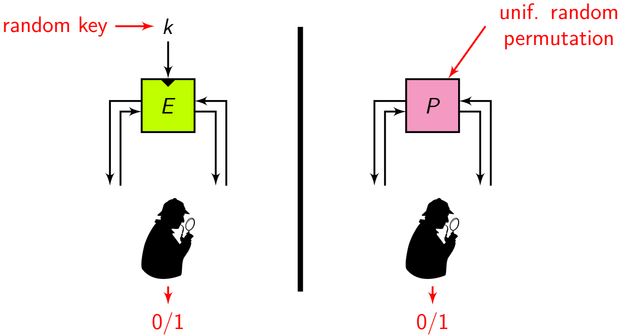

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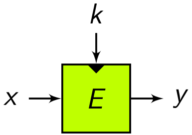

Exhaustive key search

- key k is recoverable in $\sim 2^{\kappa}$ evaluations of E

Given $\mathcal{O} \in \{P, E_k\}$:

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   (a) $y' \leftarrow E_{k'}(0^n)$
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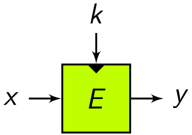

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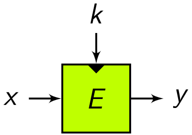

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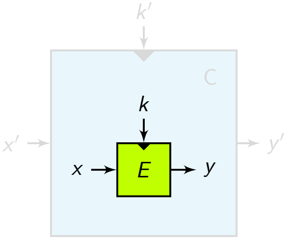
The Key-Length Extension (KLE) Problem



Goal:

construct from E a new block cipher

$$\mathrm{C}[E] : \{0,1\}^{\kappa'} \times \{0,1\}^n \to \{0,1\}^n$$

such that

- $\kappa' > \kappa$
- best generic attack requires $> 2^{\kappa}$ evaluations of E and C

Examples

- Triple Encryption
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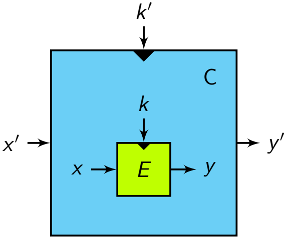

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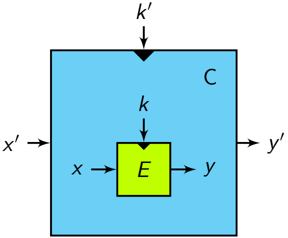

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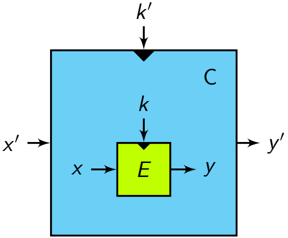

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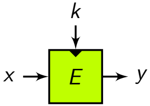
The Ideal Cipher Model (ICM)

We will model the underlying block cipher E as an ideal cipher



Ideal Block Cipher Model

- family of uniformly random permutations $E_k(\cdot)$
- independent for each key
- given as an oracle to all parties (incl. adversaries)

Generic Security

- attacks cannot exploit any weakness of E
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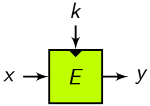

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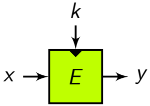

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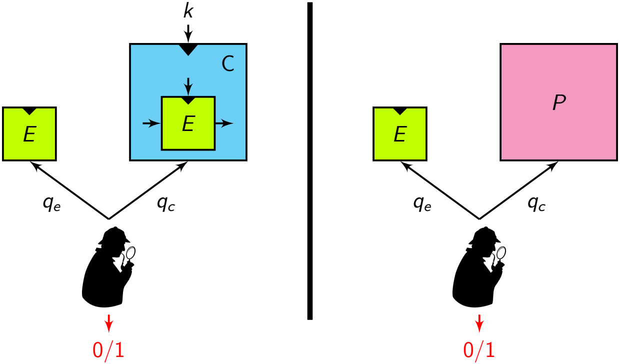

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- q_e ideal cipher queries to $E(\cdot,\cdot)$
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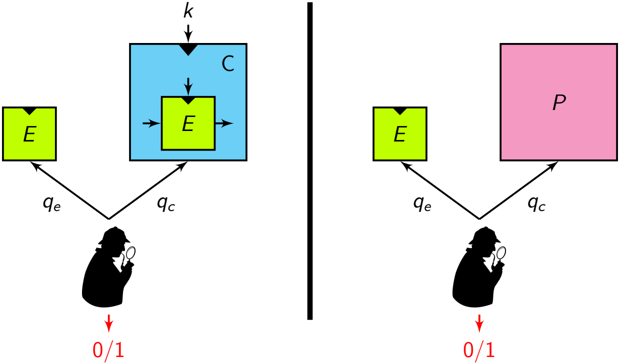

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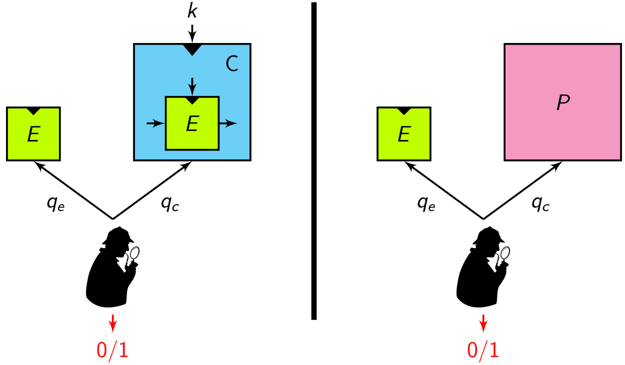

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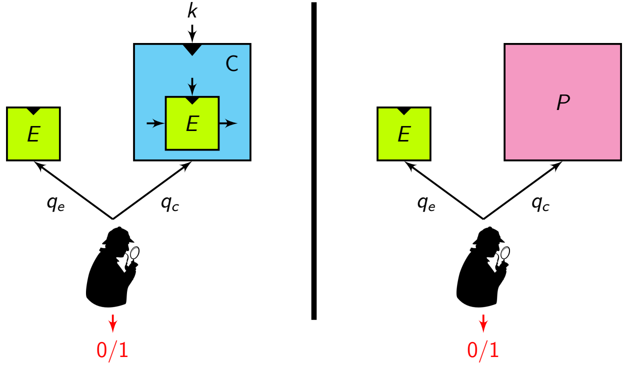

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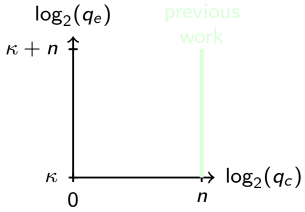
Full vs. Partial Codebook

Query Accounting

- most previous work sets $q_c = 2^n$ (*full codebook* of C[E])
  $\Rightarrow q_e$ is the only complexity measure
- too restrictive!
  - number of pt/ct pairs can be limited (frequent rekeying)
  - mode of operation may impose $q_c \ll 2^n$
- we aim at studying the entire plan (q_c, q_e)



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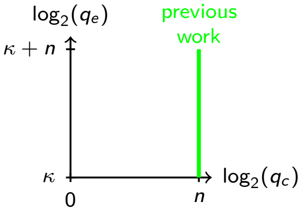

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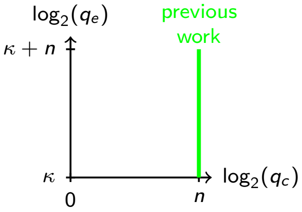

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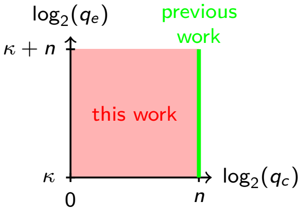

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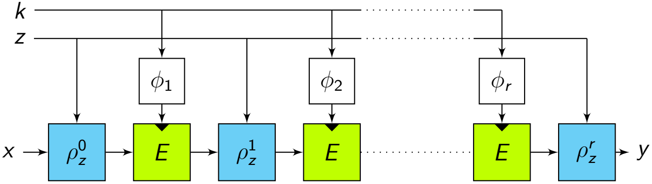
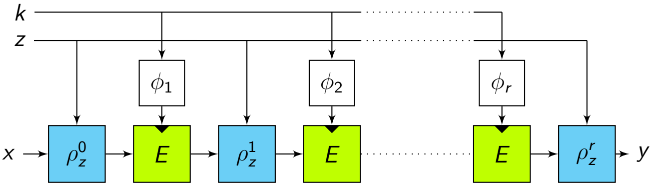


- the ρ^i's are keyed permutations, potentially very simple (e.g. $\rho_z^i(x) = x \oplus z$)
- encryption keys $\phi_1(k), \ldots, \phi_r(k)$ can be deterministically related or independent

Induced Sequential Cipher



- k fixed and known
  $\Rightarrow$ C[E] = block cipher construction using
  - independent public permutations $P_1, \ldots, P_r$
  - key z
- $\Rightarrow$ induced sequential cipher (ISC) of C, denoted $\overline{\mathrm{C}}$
- generalization of a key-alternating cipher
- well-studied design in the Random Permutation Model

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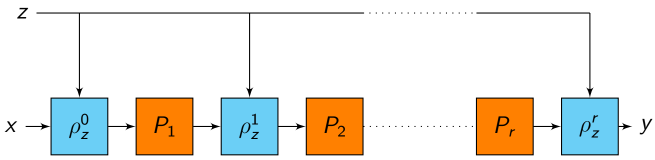

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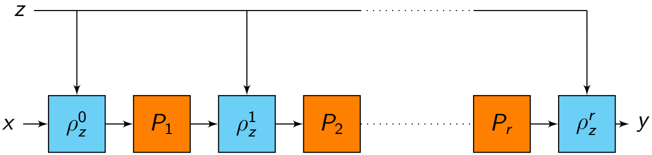

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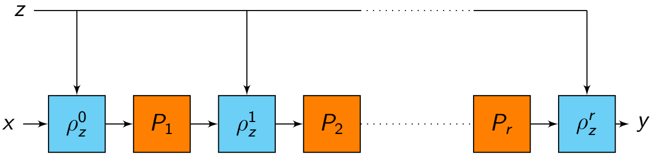

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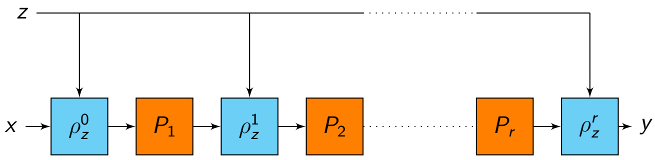

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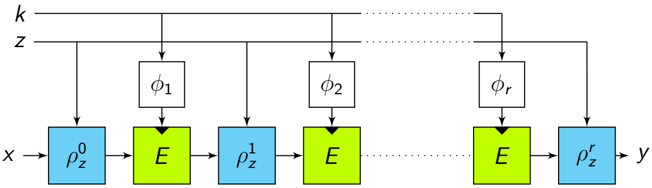
KLE-to-ISC Lemma



Allows to reduce the security analysis of a randomized KLE C to the analysis of the Induced Sequential Cipher $\overline{\mathsf{C}}$

Lemma

For any M,

Optimizing M yields a bound that depends only on q_c and q_e.

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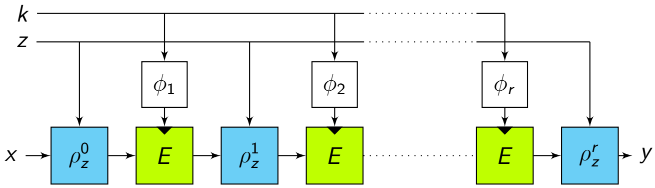

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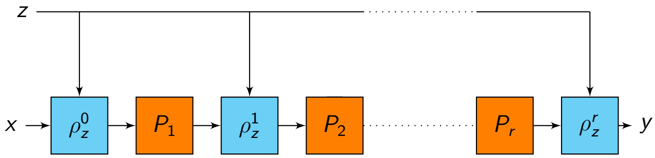

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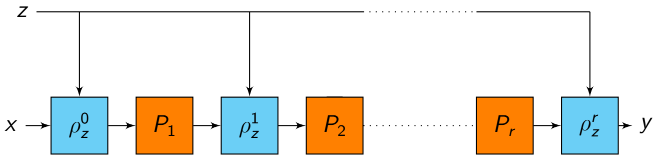
KLE-to-ISC Lemma



Allows to reduce the security analysis of a randomized KLE C to the analysis of the Induced Sequential Cipher $\overline{\mathsf{C}}$

Lemma

For any M,

$$\mathbf{Adv}_{\mathsf{C}}^{\mathrm{sprp}}(q_c, q_e) \leq \frac{rq_e}{M2^{\kappa}} + \mathbf{Adv}_{\overline{\mathsf{C}}}^{\mathrm{sprp}}(q_c, M)$$

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Outline

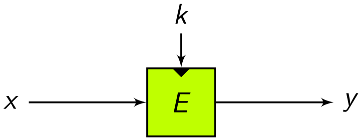
Context: Key-Length Extension for Block Ciphers

Main Lemma

Randomized Cascading

Plain Cascading

Key Whitening



FX construction (generic DESX)

- additional keys hide i./o. of E
- suggested by Rivest
- analyzed by [KR01]
- secure when $q_c \cdot q_e \ll 2^{\kappa+n}$
- same bound when $z_0 = z_1$

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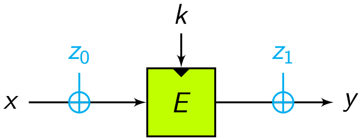

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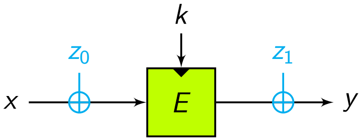

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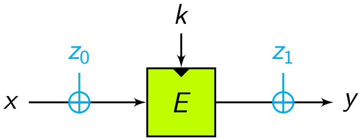

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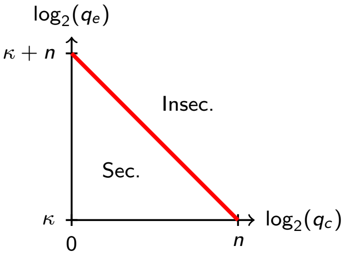

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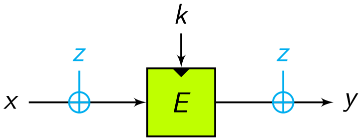

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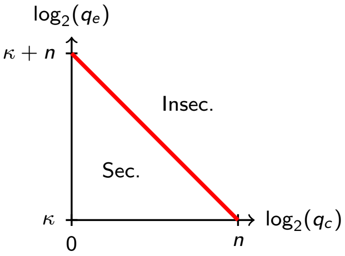

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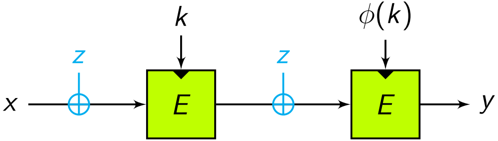

2XOR construction

- combines key-whitening and cascading
- same whitening key z
- ϕ such that $\forall k, \phi(k) \neq k$
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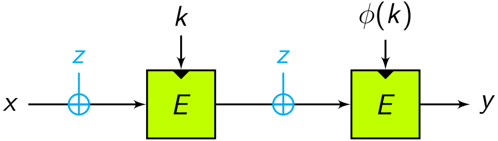

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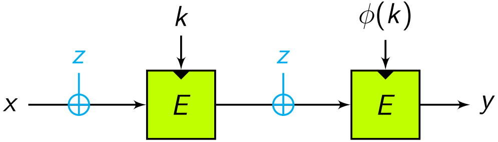

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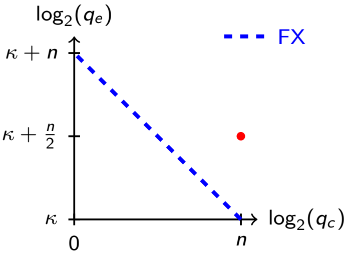
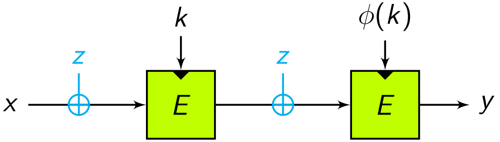


Refined Analysis of 2XOR



We (tightly) complete the picture:

- for $1 \leq q_c \leq 2^{n/2}$:
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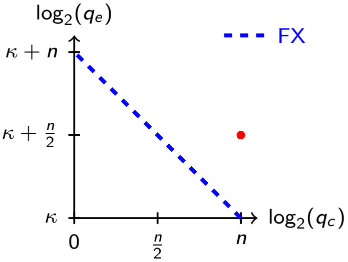

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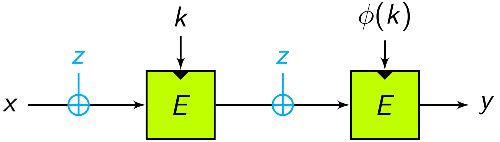

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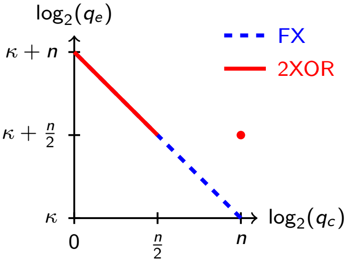

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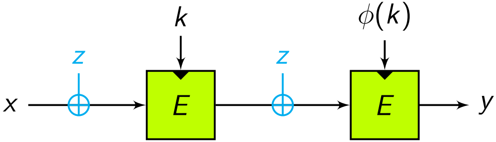

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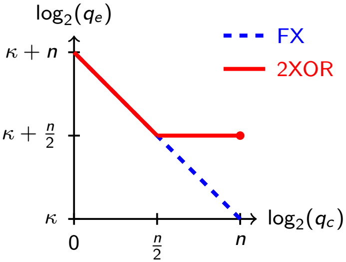
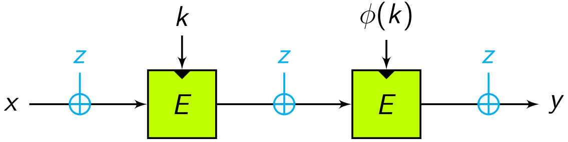


3XOR: Final Whitening Step Helps



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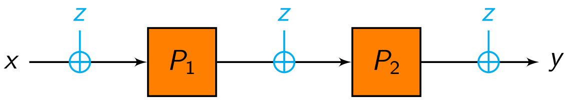

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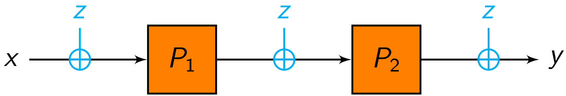

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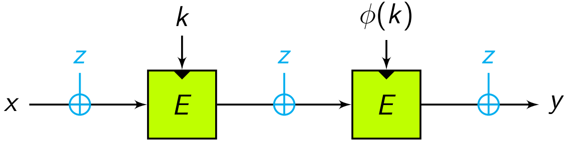

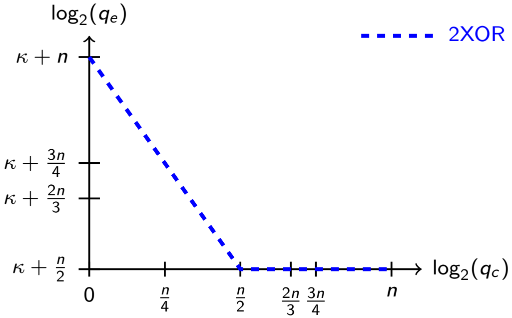

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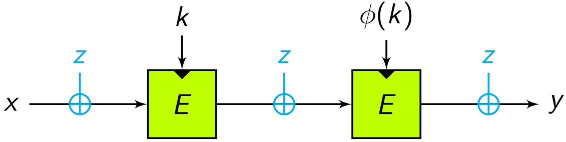

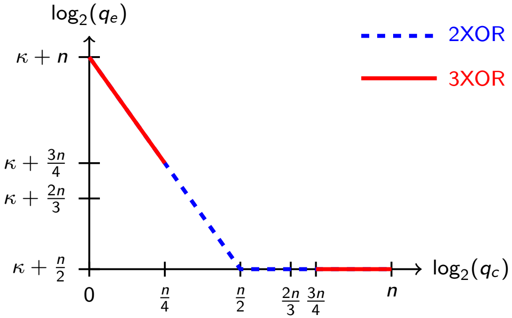

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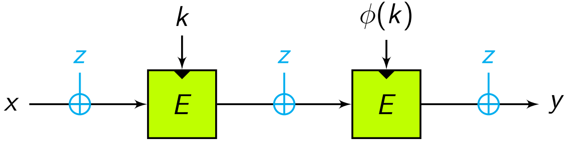

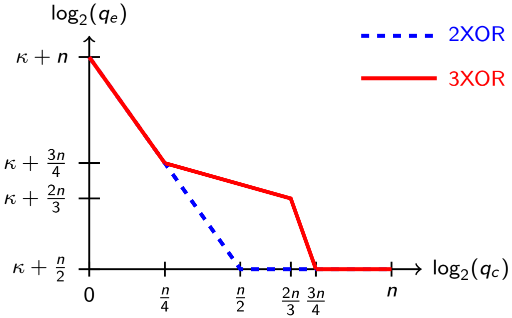

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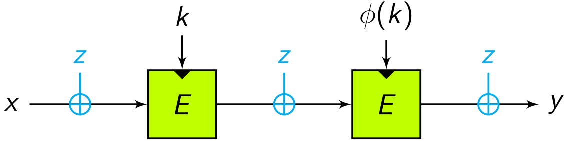

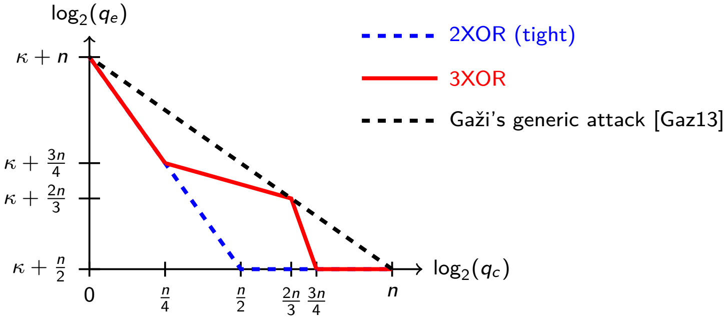

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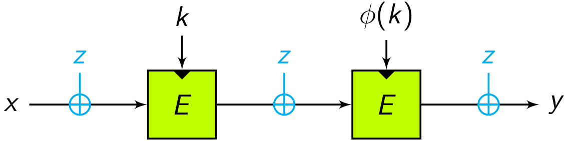
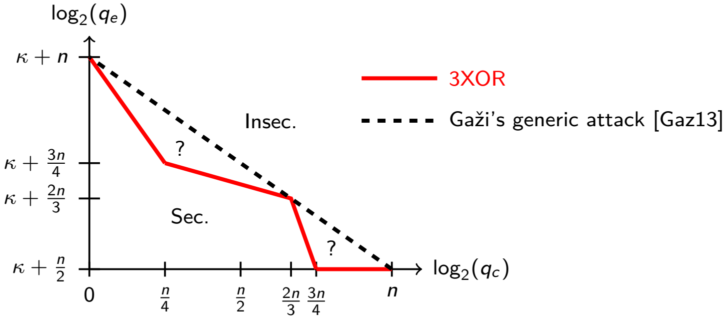
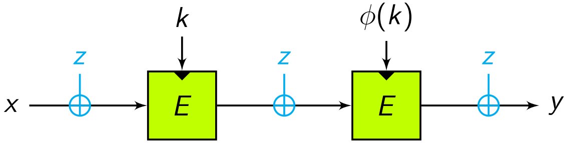




A 2-call Construction without Rekeying



- drawback of 2XOR and 3XOR constructions: call the block cipher E with two distinct keys
- we propose a construction calling E twice with the same key
- π is a linear orthomorphism
- security bound qualitatively similar to 3XOR

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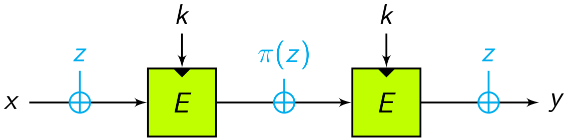

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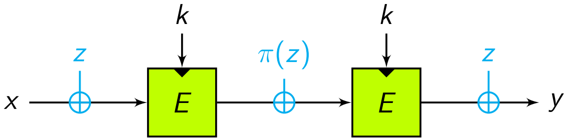

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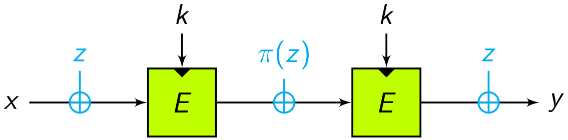

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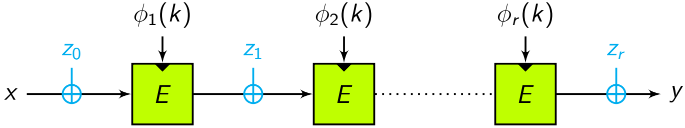

Xor-Cascade Encryption: XCE

- independent whitening keys, distinct encryption keys
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- r-round XCE is secure as long as $q_c \cdot q_e^r \ll 2^{r(\kappa+n)}$
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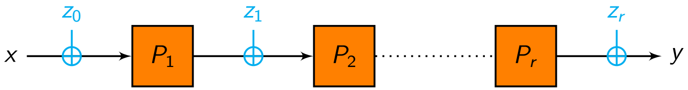

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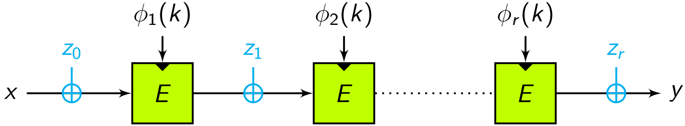

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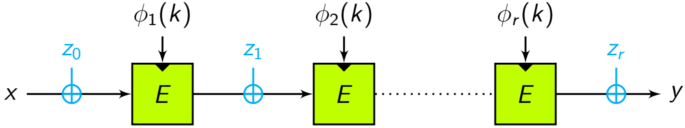

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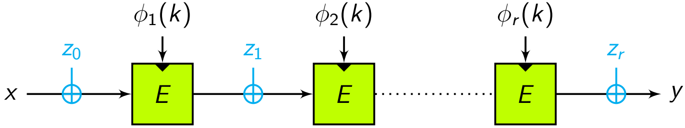

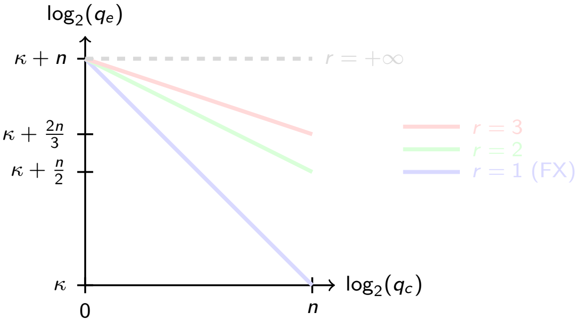

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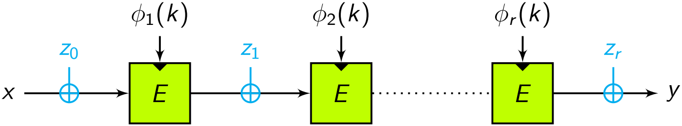

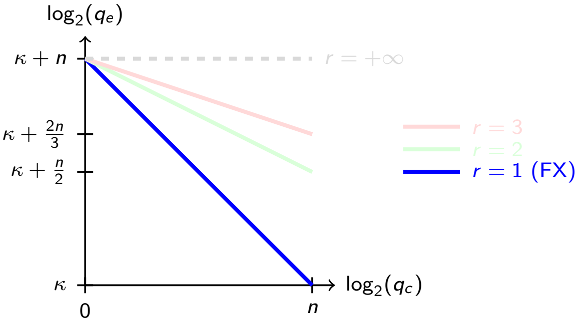

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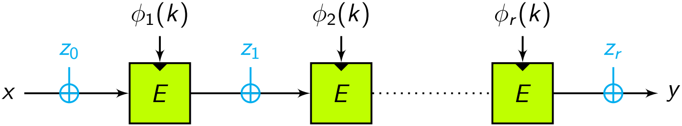

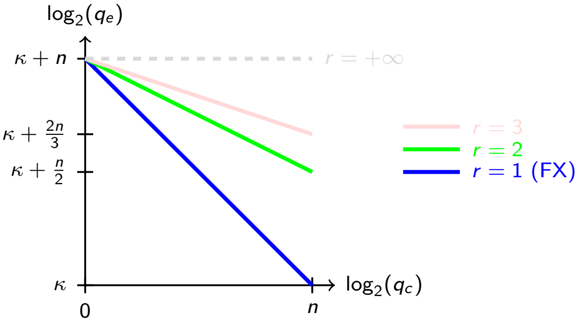

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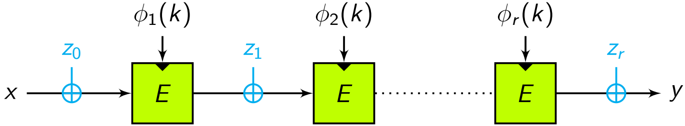

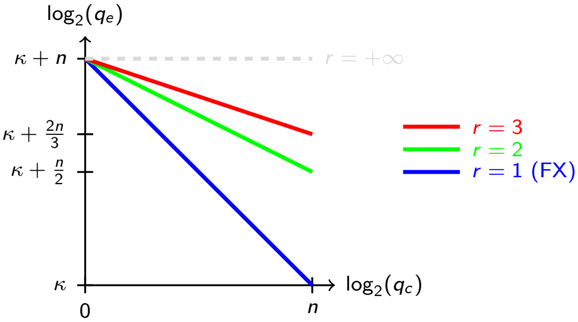

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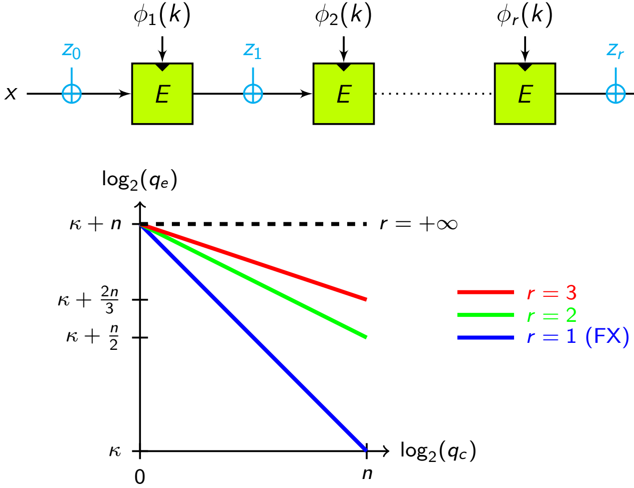

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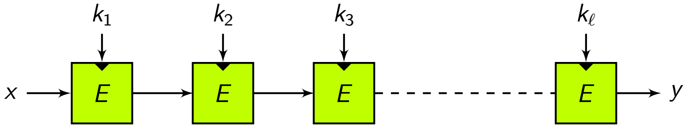

Cascade Encryption

- encrypt ℓ times with independent keys
- $\ell = 2$ does not help (meet-in-the-middle attack [DH77])
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- tight bound for $q_c = 2^n$ [DLMS14]: for odd ℓ, secure when

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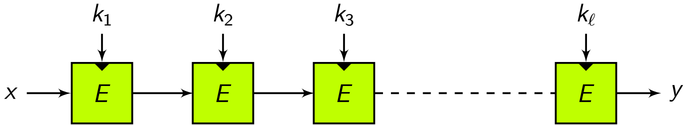

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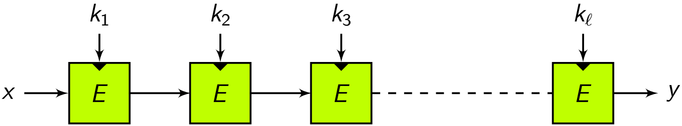

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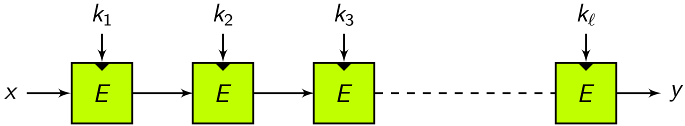

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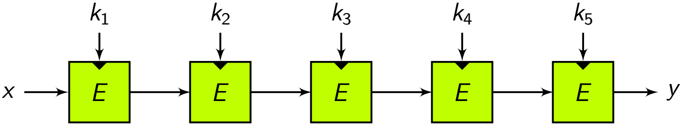
Our Analysis of Plain Cascade Encryption



- use 2 independent ideal ciphers E, E' (key-domain separation)
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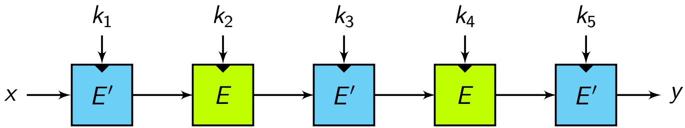

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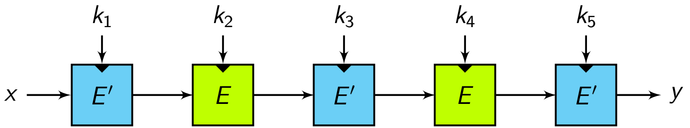

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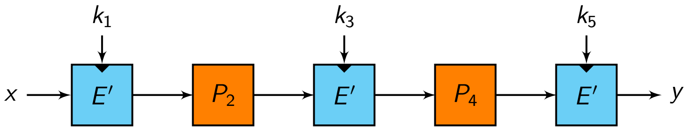

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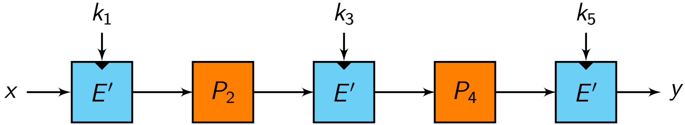

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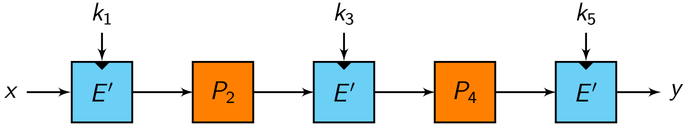

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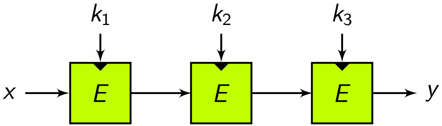

- our bound:

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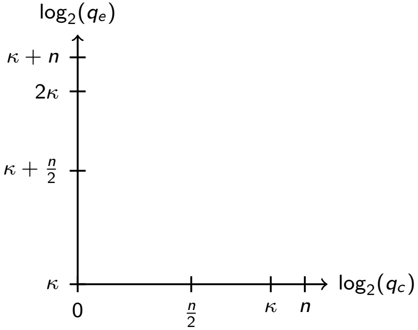

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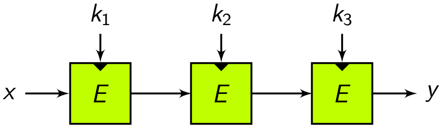

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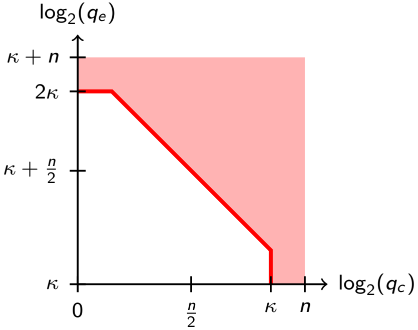

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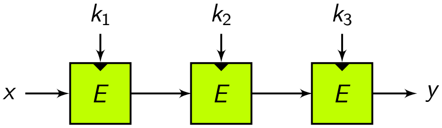

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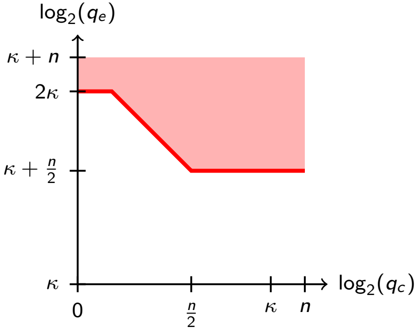

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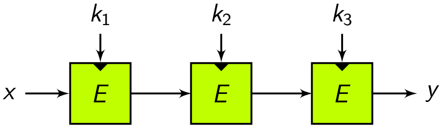

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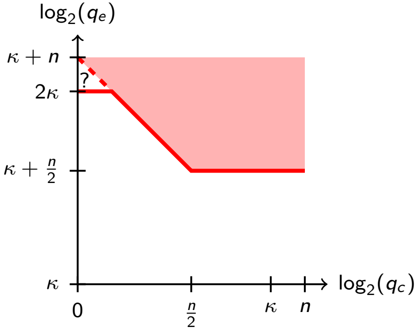
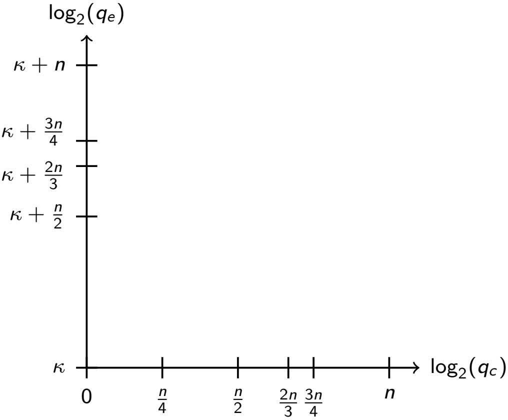


Conclusion I



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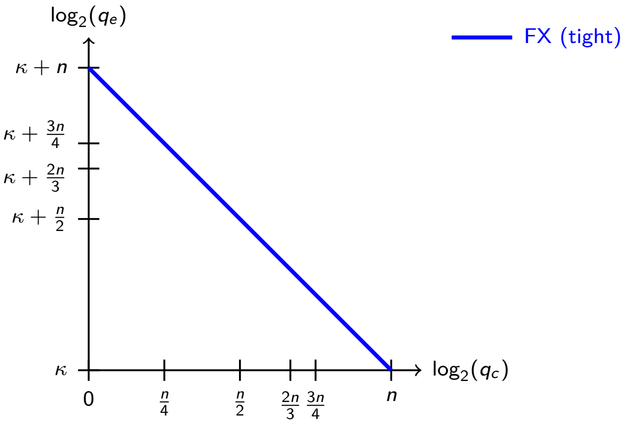

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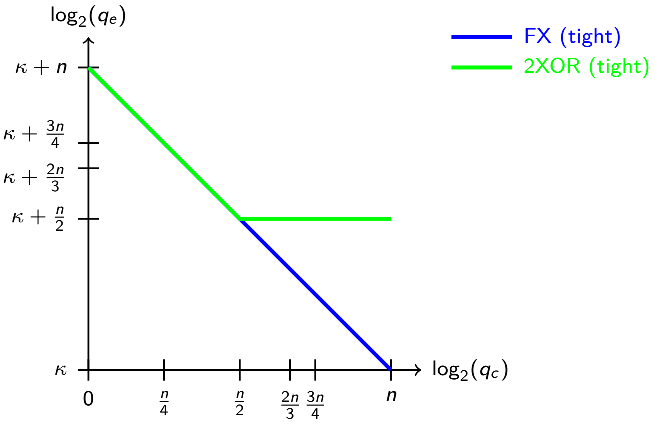

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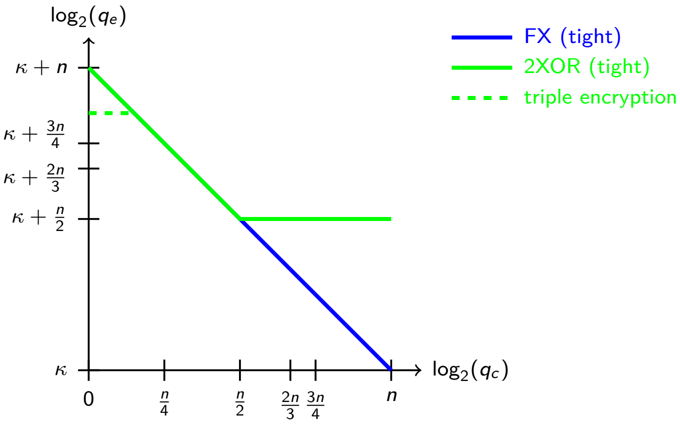

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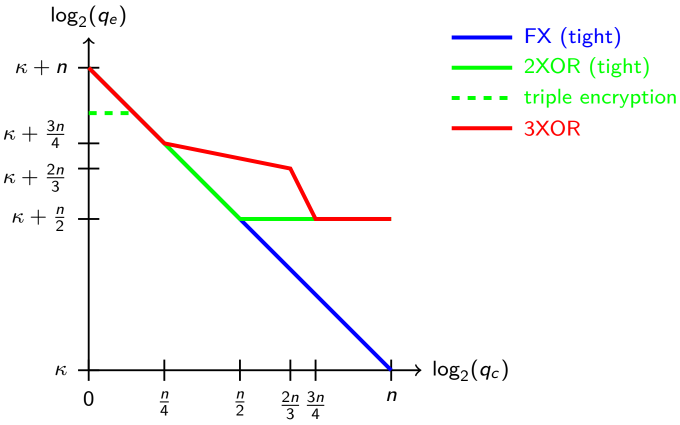

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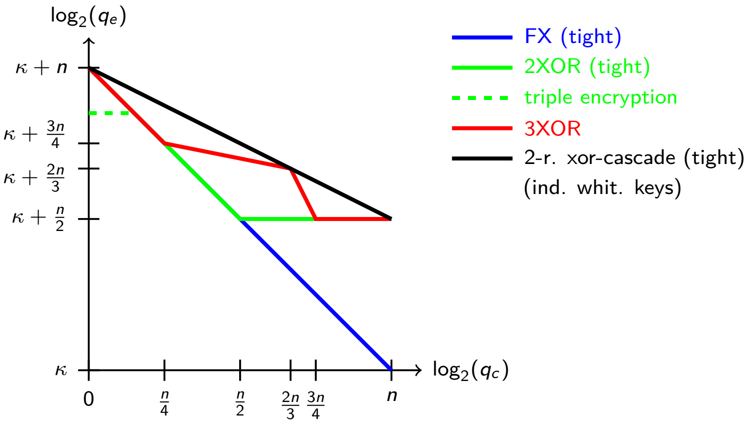

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The end...

Thanks for your attention!

Comments or questions?

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