

# A Survey of Recent Results on Key-Alternating Ciphers

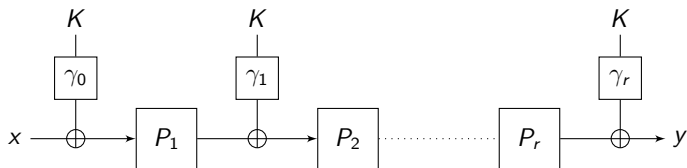
Yannick Seurin  
(based on joint work with  
R. Lampe and J. Patarin)

ANSSI

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# Introduction

A key-alternating cipher with  $r$  rounds is the following construction:

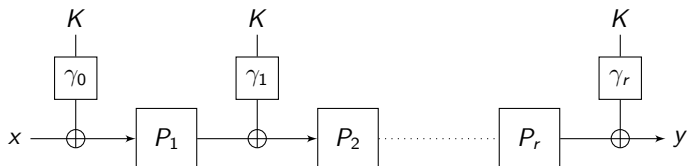


- The  $P_i$ 's are public permutations on  $\{0, 1\}^n$
- $K \in \{0, 1\}^\ell$  is the (master) key
- The  $\gamma_i$ 's are key derivation functions mapping  $K$  to  $n$ -bit values

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Most (if not all) SPN ciphers can be described as key-alternating ciphers. E.g. for AES-128, one has  $r = 10$ , the  $\gamma_i$ 's are efficiently invertible permutations, and:

$$P_1 = \dots = P_9 = \text{SubBytes} \circ \text{ShiftRows} \circ \text{MixColumns}$$

$$P_{10} = \text{SubBytes} \circ \text{ShiftRows}$$

When the  $P_i$ 's are fixed permutations, one can prove results like:

- the best differential characteristic over  $r' < r$  rounds has probability at most  $p$
- the best linear approximation over  $r' < r$  rounds has probability at most  $p'$

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Recently, a lot of results have been obtained in the **Random Permutation Model**: the  $P_i$ 's are viewed as **oracles** to which the adversary can make black-box queries (both to  $P_i$  and  $P_i^{-1}$ ).

Interpretation: gives a guarantee against **any** adversary which do not use particular properties of the  $P_i$ 's

In fact, this model was already considered 15 years ago by Even and Mansour for  $r = 1$  round: they showed that the following cipher is secure up to  $\mathcal{O}(2^{n/2})$  queries of the adversary:

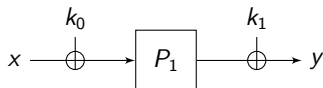


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  - Introduction
  - The coupling technique
  - The indistinguishability proof
- 2 Interlude: tweakable block ciphers
- 3 Indifferentiability
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  - Indifferentiability of the IEM cipher
  - At least 4 rounds are necessary
  - Indifferentiability proof for 12 rounds



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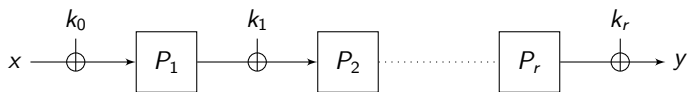
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# The IEM cipher with independent keys

We focus in this part on the IEM cipher **with independent round keys**:

$$K = (k_0, k_1, \dots, k_r)$$

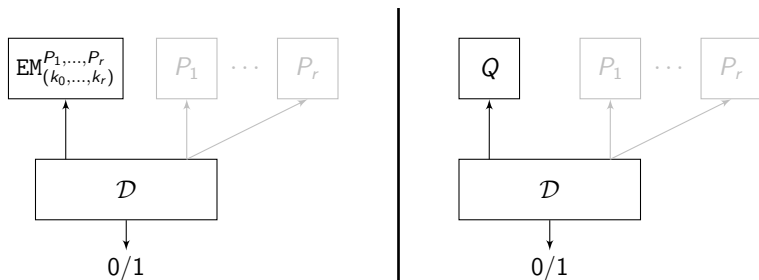


Total key space:  $\{0, 1\}^{(r+1)n}$

Notation:

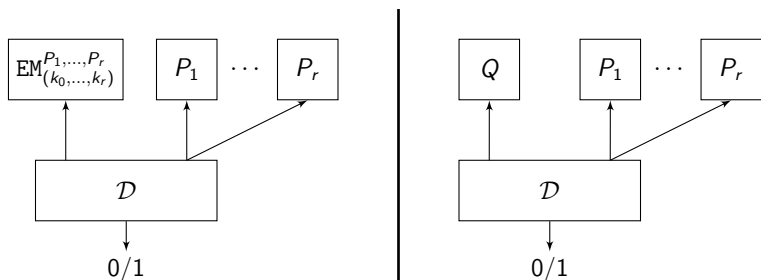
$$y = \text{EM}_{(k_0, \dots, k_r)}^{P_1, \dots, P_r}(x) .$$

# Formalizing indistinguishability for the IEM cipher



- left:  $k_0, \dots, k_r \leftarrow_{\$} \{0, 1\}^n$  are randomly chosen keys
- right:  $Q$  is a random permutation independent of  $P_1, \dots, P_r$
- we are in the Random Permutation Model: the distinguisher also has oracle access to  $P_1, \dots, P_r$  in both worlds

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# Indistinguishability of the IEM cipher: Summary of results

Results for **independent round keys**  $(k_0, k_1, \dots, k_r)$

Notation:  $N = 2^n$

- for  $r = 1$  round, EM is secure up to  $\mathcal{O}(N^{1/2})$  queries [EM97]
- for  $r \geq 2$ , EM is secure up to  $\mathcal{O}(N^{2/3})$  queries [BKL<sup>+</sup>12]
- for any even  $r$ , EM is secure up to  $\mathcal{O}(N^{r/(r+2)})$  queries [LPS12]
- **tight result**: EM is secure up to  $\mathcal{O}(N^{r/(r+1)})$  queries [CS14]

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## Coupling: definition

### Definition (Coupling)

Let  $\mu$  and  $\nu$  be two probability distributions on  $\Omega$ . A **coupling** of  $\mu$  and  $\nu$  is a probability dist.  $\lambda$  on  $\Omega \times \Omega$  such that:

$$\forall x \in \Omega, \sum_{y \in \Omega} \lambda(x, y) = \mu(x)$$

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In other words,  $\lambda$  is a joint probability distribution whose marginal distributions are resp.  $\mu$  and  $\nu$ .

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## Lemma

Let  $\mu$  and  $\nu$  be two probability distributions and  $\lambda$  be a coupling. Let  $(X, Y) \sim \lambda$ . Then:

$$\|\mu - \nu\| \leq \Pr[X \neq Y] .$$

- Introduced by Aldous, key tool to study the mixing time of Markov chains
- First used in crypto by Mironov [Mir02] to analyze the shuffle of RC4, later by [MRS09, HR10] to analyze Feistel ciphers

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## A (very) simple example

Two couplings of the uniform distribution on  $\{1, 2, 3, 4\}$  with itself:

X/Y	1	2	3	4
1	1/16	1/16	1/16	1/16
2	1/16	1/16	1/16	1/16
3	1/16	1/16	1/16	1/16
4	1/16	1/16	1/16	1/16

$$\Pr[X \neq Y] = 3/4$$

X/Y	1	2	3	4
1	1/4	0	0	0
2	0	1/4	0	0
3	0	0	1/4	0
4	0	0	0	1/4

$$\Pr[X \neq Y] = 0$$

Not all couplings give good upper bounds on  $\|\mu - \nu\|$

NB: there always exists a coupling  $\lambda$  for which equality

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Two coins:

- a perfect one:  $p_{\text{head}} = 0.5$
- a biased one:  $p'_{\text{head}} = 0.6$

Show that over  $N$  tosses, the probability that the biased coin makes  $k$  heads is larger than the probability that the perfect coin makes  $k$  heads (for any  $k \leq N$ ). Two solutions:

- 1 compute the binomial law: a bit tedious. . .
  - 2 couple the two distributions as follows:
    - toss the perfect coin
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## Two types of distinguishers

NB:  $\mathcal{D}$  is computationally unbounded and makes at most  $q$  queries to each oracle

We define the two following classes of distinguishers:

- **NCPA** (Non-Adaptive Chosen Plaintext Attacks):
  - works in two phases:
    - $\mathcal{D}$  first queries  $P_1, \dots, P_r$  as it wishes (in both directions, adaptively);
    - then it makes  $q$  non-adaptive direct queries to  $EM^{P_1, \dots, P_r}/Q$
- **CCA** (Chosen Ciphertext Attacks):
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# The case of NCPA distinguishers: the result

We will show the following:

## Theorem

*For any NCPA  $\mathcal{D}$  making at most  $q$  queries to each oracle, the distinguishing advantage against the IEM with  $r$  rounds is at most*

$$2^r \frac{q^{r+1}}{N^r} .$$

→ security up to  $\mathcal{O}(N^{r/(r+1)})$  queries.

# The case of NCPA distinguishers: a matching attack

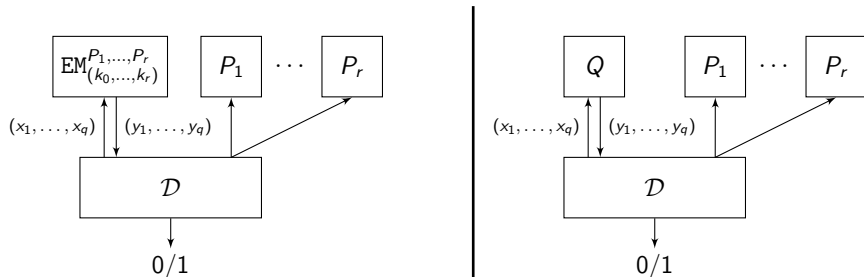
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A matching attack has been described in [BKL<sup>+</sup>12]:

- make  $\mathcal{O}(N^{r/(r+1)})$  queries to the cipher and to each  $P_i$
- for each possible key, find a “contradictory path”
- any wrong key will have a contradictory path with high proba.
- (note: this is just exhaustive key search, but we are interested in the number of queries rather than computational cost)



# The case of NCPA distinguishers

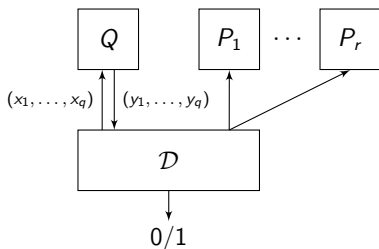
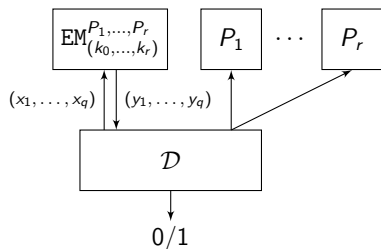


$\mathcal{D}$  first makes  $q$  queries to  $P_1, \dots, P_r$  and obtains equations:

$$P_i(a_{i,j}) = b_{i,j}, \quad i \in [1, r], \quad j \in [1, q].$$

Then it makes  $q$  non-adaptive queries  $(x_1, \dots, x_q)$  to  $EM/Q$  and receives answers  $(y_1, \dots, y_q)$

# The case of NCPA distinguishers



The distribution of  $(a_{i,j}), (b_{i,j})$  is the same in both worlds  
 $\rightarrow$  the advantage of  $\mathcal{D}$  is given by the statistical distance between the distributions of  $(y_1, \dots, y_q)$  in the real and the ideal world

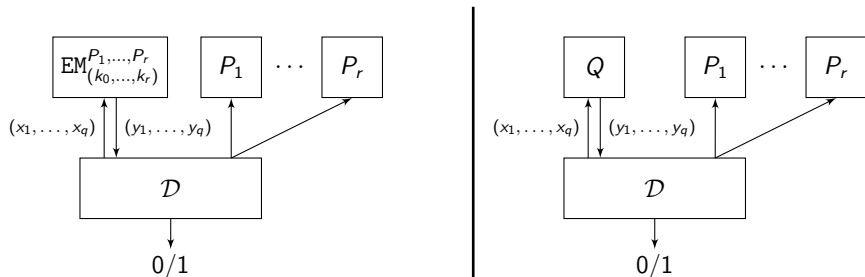
Notation:

$\mu_q$  = distribution of  $(y_0, \dots, y_q)$  in the real world

$\mu_0$  = distribution of  $(y_0, \dots, y_q)$  in the ideal world (uniform)

$\rightarrow$  we want to upper bound  $\|\mu_q - \mu_0\|$

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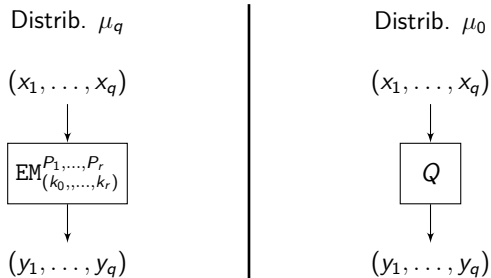
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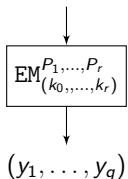
The distribution  $\mu_q$  in the real world is obtained as follows:

- draw random permutations  $P_1, \dots, P_r$  satisfying  $P_i(a_{i,j}) = b_{i,j}$
- draw independent random round keys  $(k_0, \dots, k_r)$
- let  $y_i = \text{EM}_{(k_0, \dots, k_r)}^{P_1, \dots, P_r}(x_i)$

# A hybrid argument

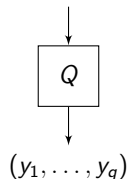
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Distrib.  $\mu_0$

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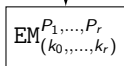


The uniform distribution  $\mu_0$  is also obtained by drawing uniformly random (distinct) inputs  $(u_1, \dots, u_q)$  and computing their image through EM

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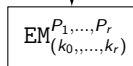
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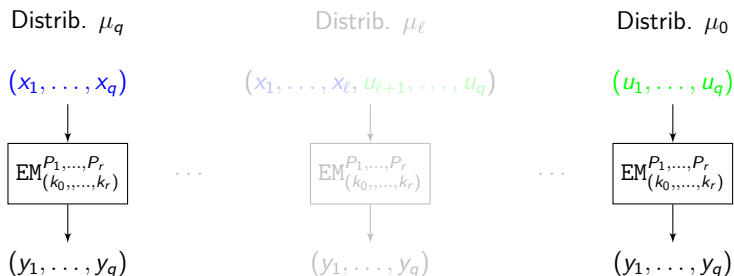
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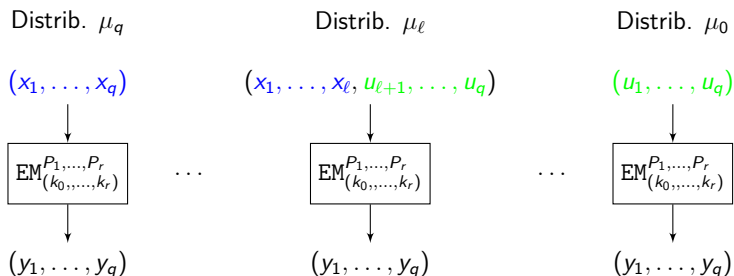


Hybrid distributions  $\mu_\ell$ ,  $\ell \in [0, q]$

$$\|\mu_q - \mu_0\| \leq \sum_{\ell=0}^{q-1} \|\mu_{\ell+1} - \mu_\ell\| .$$

→ We will upper bound  $\|\mu_{\ell+1} - \mu_\ell\|$  with a coupling.

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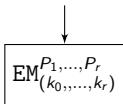
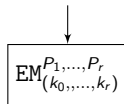


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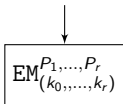
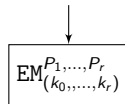
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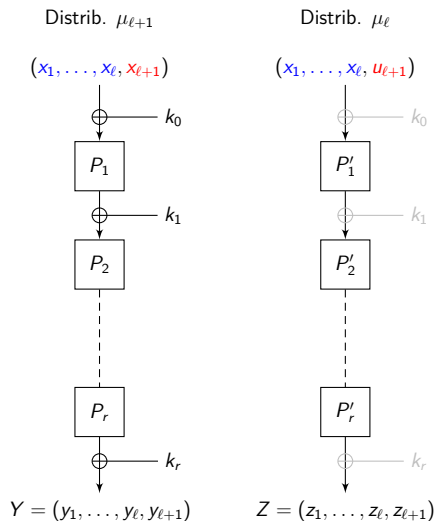


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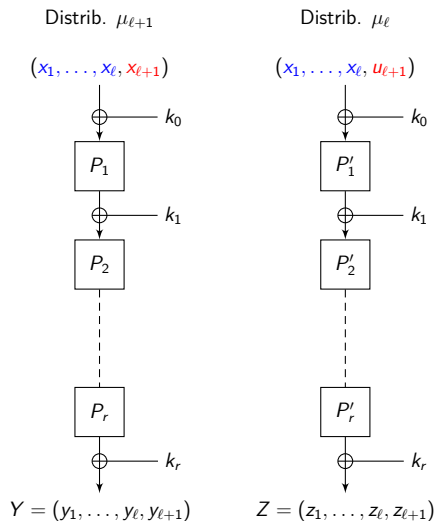
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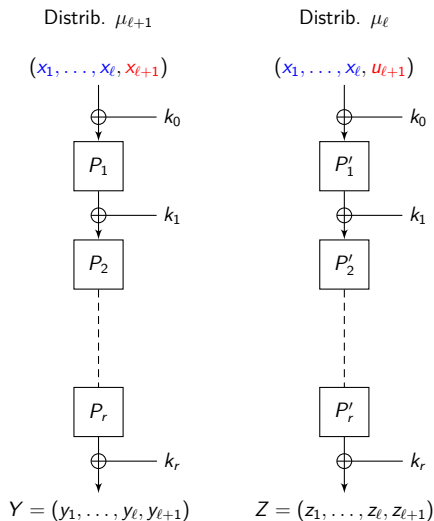
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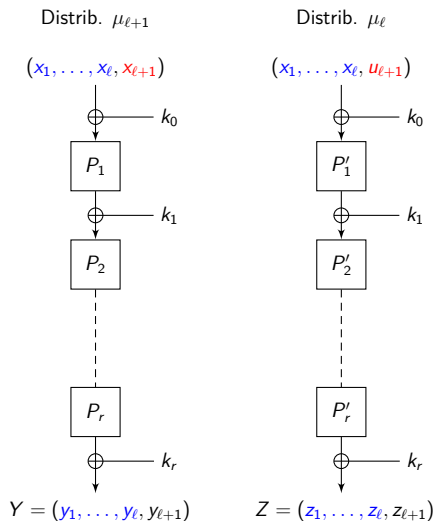
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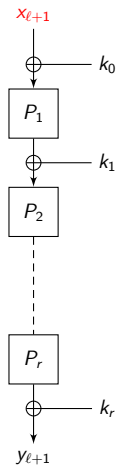
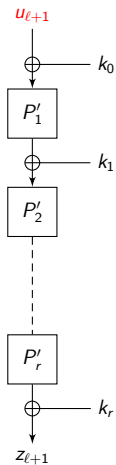
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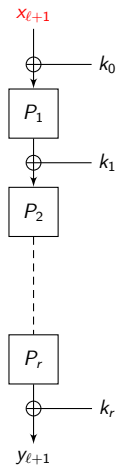
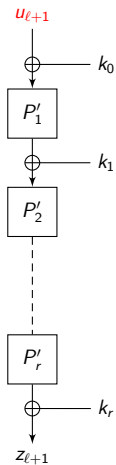
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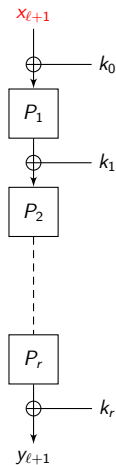
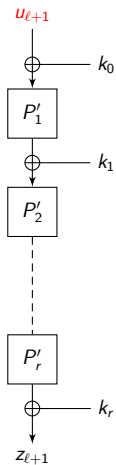
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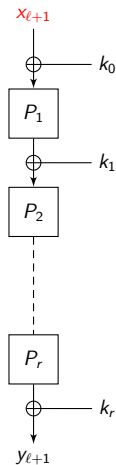
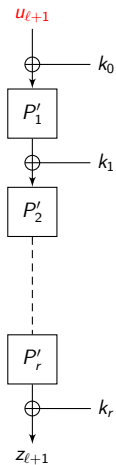
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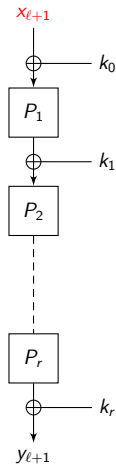
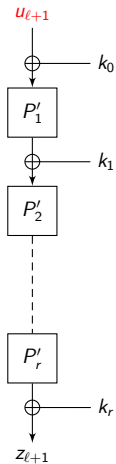


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- we proceed iteratively for  $i = 1..r$  as follows:

- if  $u_{\ell+1}^i$  is not free, then  $P'_i(u_{\ell+1}^i)$  is imposed by the equations  $P'_i(a_{i,j}) = b_{i,j}$
- if  $u_{\ell+1}^i$  is free but  $x_{\ell+1}^i$  is not, we define  $P'_i(u_{\ell+1}^i)$  uniformly at random among possible values
- if  $u_{\ell+1}^i$  and  $x_{\ell+1}^i$  are both free, we define

$$P'_i(u_{\ell+1}^i) = P_i(x_{\ell+1}^i)$$

→ successful coupling, the subsequent outputs remain equal

## Coupling $\mu_{\ell+1}$ and $\mu_\ell$

We have  $Y \neq Z$  only if we fail to couple at all rounds  $i = 1, \dots, r$ .

Probability to fail to couple at round  $i$

(given that it failed at rounds  $1, \dots, i-1$ ):

Since  $x_{\ell+1}^i$  and  $u_{\ell+1}^i$  are randomized by key  $k_{i-1}$ , and since  $|(a_{i,j})| = q$ , the probability that  $x_{\ell+1}^i$  or  $u_{\ell+1}^i$  is not free is at most  $2q/N$ .

Hence, the probability to fail to couple at all  $r$  rounds and to have  $Y \neq Z$  at the output of the two EM ciphers is:

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By the coupling lemma

$$\|\mu_{\ell+1} - \mu_\ell\| \leq \Pr[Y \neq Z] \leq \left(\frac{2q}{N}\right)^r .$$

Hence:

$$\|\mu_q - \mu_0\| \leq \sum_{\ell=0}^{q-1} \|\mu_{\ell+1} - \mu_\ell\| \leq 2^r \frac{q^{r+1}}{N^r} .$$

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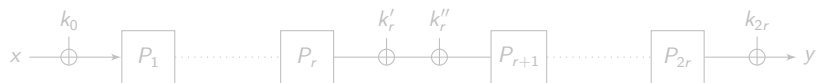
# From NCPA to CCA security

We use the following “two weak make one strong” composition theorem:

**Theorem ([MPR07])**

*Let  $E$  and  $F$  be two NCPA-secure block ciphers, with the same domain and resp. key spaces  $\mathcal{K}_E$  and  $\mathcal{K}_F$ . Then  $E \circ F^{-1}$  is a CCA-secure block cipher with key space  $\mathcal{K}_E \times \mathcal{K}_F$ .*

The IEM cipher with  $2r$  rounds is the composition of 2 IEM ciphers with  $r$  rounds (splitting the key  $k_r = k'_r \oplus k''_r$ ):



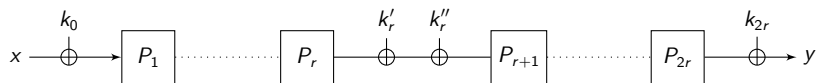
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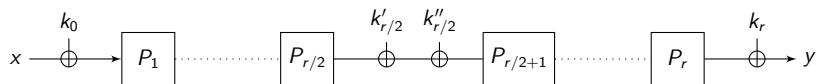
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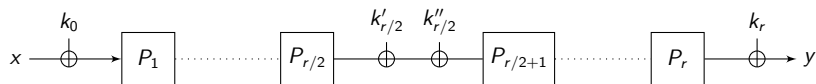
For any CCA  $\mathcal{D}$  making at most  $q$  queries to each oracle, the distinguishing advantage against the IEM with  $r$  rounds ( $r$  even) is at most

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→ security up to  $\mathcal{O}(N^{r/(r+2)})$  queries.

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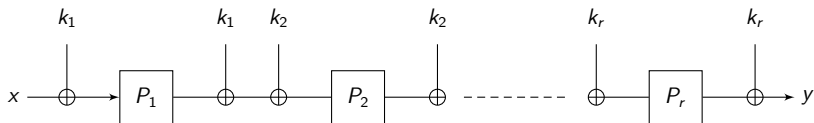
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# Extensions and open problems

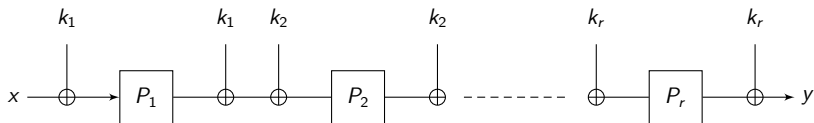
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# Tweakable block ciphers: definition

A tweakable block cipher (TBC) is a family of block ciphers indexed by a tweak  $t \in \mathcal{T}$ :

$$\tilde{E} : \mathcal{T} \times \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{M}$$

The tweak is a **public** parameter (under the control of the adversary in the security model)

Introduced by Liskov, Rivest, and Wagner at CRYPTO 2002 [LRW02].



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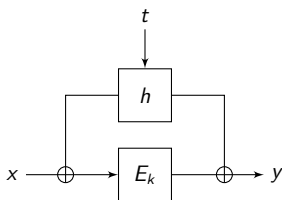
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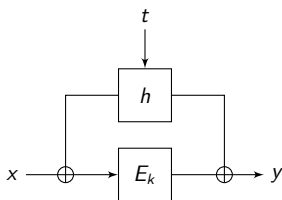


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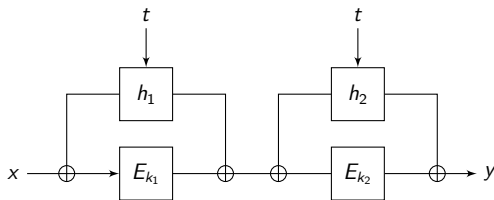


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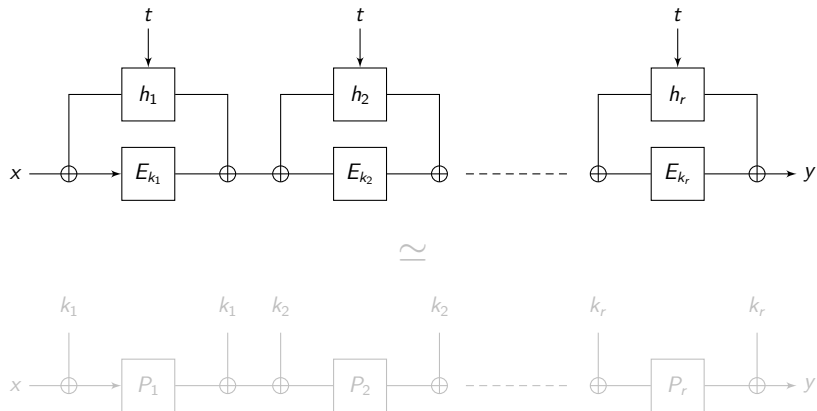
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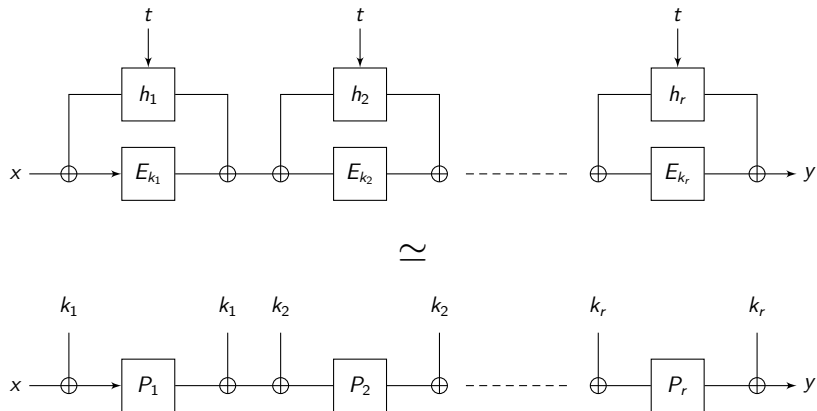
# The [LST12] construction

At CRYPTO 2012, Landecker et al. extended the LRW construction as follows:

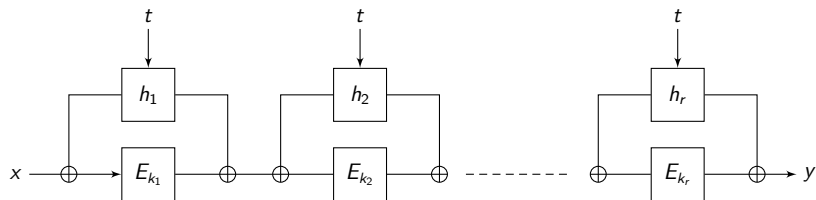


[LST12] proved security (against CCA adversaries) up to  $\mathcal{O}(2^{2n/3})$  queries.

Extension to  $r$  rounds

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# Extension to $r$ rounds



For this TBC construction, one can prove results similar to the ones for the IEM cipher [LS13]:

- secure against NCPA distinguishers up to  $\mathcal{O}(2^{rn/(r+1)})$  queries
- secure against CCA distinguishers up to  $\mathcal{O}(2^{rn/(r+2)})$  queries

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Previous results state that the IEM cipher is a (strong) pseudorandom permutation (in the random permutation model)

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What about related-, known- or chosen-key attacks?

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- the pseudorandomness security notion for a block cipher is sufficient to prove the security of a lot of applications (encryption modes and MACs)
- however, sometimes it is not sufficient (e.g. for block cipher-based hash functions like Davies-Meyer mode)
- ideally, one expects that a good block cipher “behaves” as an independent random permutation for each key  
→ **ideal cipher model**
- similar to the random oracle model for a hash function  
warning: instantiation problems as well (no concrete block cipher can be proved to be an ideal cipher in any reasonable sense)
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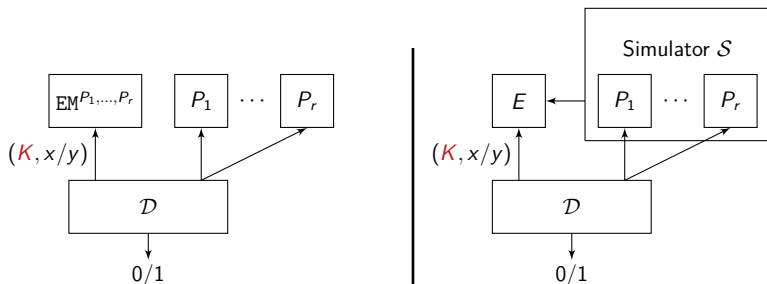
## A word on the ideal cipher model

- the pseudorandomness security notion for a block cipher is sufficient to prove the security of a lot of applications (encryption modes and MACs)
- however, sometimes it is not sufficient (e.g. for block cipher-based hash functions like Davies-Meyer mode)
- ideally, one expects that a good block cipher “behaves” as an independent random permutation for each key  
→ **ideal cipher model**
- similar to the random oracle model for a hash function  
warning: instantiation problems as well (no concrete block cipher can be proved to be an ideal cipher in any reasonable sense)
- though we cannot prove that a block cipher behaves as an ideal cipher in the standard model, we can prove results in **idealized models** (e.g. the Random Permutation Model that we already used for the IEM cipher)  
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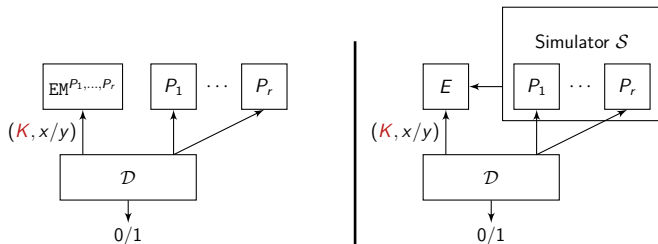
# Indifferentiability: definition

## Definition

A construction  $\mathcal{C}^F$  (here, the IEM cipher  $\text{EM}^{P_1, \dots, P_r}$ ) using an ideal primitive  $F$  (here, random permutations  $P_1, \dots, P_r$ ) is said indifferentiable from an ideal primitive  $G$  (here, an ideal cipher  $E$ ) if there exists a polynomial time simulator  $S$  with access to  $G$  such that the two systems  $(\mathcal{C}^F, F)$  and  $(G, S^G)$  are indistinguishable.



# Indifferentiability: definition



The answers of the simulator  $\mathcal{S}$  must be:

- **coherent** with answers the distinguisher can obtain directly from  $E$
- **close in distribution** to the answers of a random permutation

NB: The distinguisher specifies the key and the plaintext/ciphertext when querying  $EM^{P_1, \dots, P_r}$  or  $E$ .

# Composition theorem

Usefulness of indifferentiability: composition theorem

## Theorem

*If a cryptosystem  $\Gamma$  is secure when used with an ideal primitive  $\mathbf{G}$ , and if  $\mathcal{C}^{\mathbf{F}}$  is indifferentiable from  $\mathbf{G}$ , then  $\Gamma$  is also secure when used with  $\mathcal{C}^{\mathbf{F}}$ .*

Sketch of the proof:

- assume  $\mathcal{C}^{\mathbf{F}}$  is indifferentiable from  $\mathbf{G}$
- assume there is an attacker  $\mathcal{A}$  with advantage  $\varepsilon$  against some cryptosystem  $\Gamma$  using the construction  $\mathcal{C}^{\mathbf{F}}$
- then one can consider the simulator  $\mathcal{S}$  ensured by indifferentiability
- combining  $\mathcal{A}$  and  $\mathcal{S}$ , one obtains a new attacker  $\mathcal{A}'$  against cryptosystem  $\Gamma$  used with  $\mathbf{G}$  with advantage  $\simeq \varepsilon$ , a contradiction

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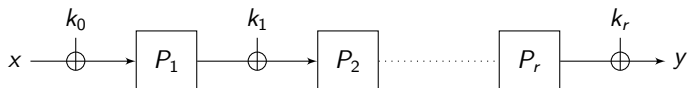
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  - At least 4 rounds are necessary
  - Indifferentiability proof for 12 rounds

# Independent round keys fails



This is not indifferentiable from an ideal cipher with key space  $\{0, 1\}^{(r+1)n}$  because of the following distinguisher:

- fix a non-zero constant  $c \in \{0, 1\}^n$
- choose an arbitrary  $x \in \{0, 1\}^n$  and  $k_0 \in \{0, 1\}^n$
- define  $x' = x \oplus c$  and  $k'_0 = k_0 \oplus c$
- let  $K = (k_0, k_1, \dots, k_r)$  and  $K' = (k'_0, k_1, \dots, k_r)$
- then  $\text{EM}(K, x) = \text{EM}(K', x')$
- this holds only with negligible probability for an ideal cipher

# Proving indifferentiability for key-alternating ciphers

Independent keys leave too much “freedom” to the adversary.

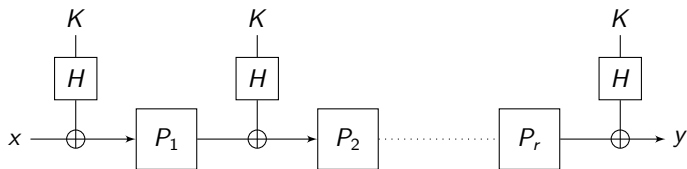
Two ideas to solve the problem:

- 1 add a key schedule, and put some cryptographic assumption on it  
⇒ Andreeva et al. CRYPTO 2013 [ABD<sup>+</sup>13]
- 2 restrain the key space and correlate the round keys, e.g.  $(k, k, \dots, k)$   
⇒ Lampe and Seurin 2013 (preprint)



# The [ABD<sup>+</sup>13] result

The key-derivation function is modeled as a **random oracle** from  $\{0, 1\}^\ell$  to  $\{0, 1\}^n$  (that the adversary queries in a black-box way)

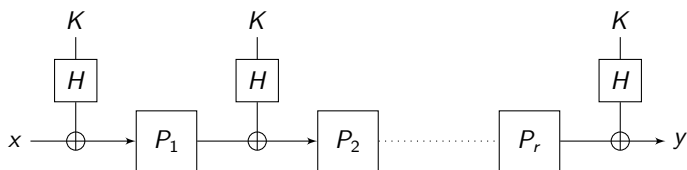


→ indifferentiable from an ideal cipher with  $\ell$ -bit keys for  $r = 5$   
 ([ABD<sup>+</sup>13] gives attacks up to 3 rounds)

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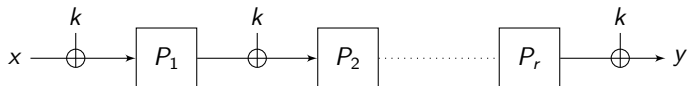


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# Our approach

We consider the IEM with a single key:



The trivial attack on independent keys does not apply  $\rightarrow$  is it indiff. from an ideal cipher for sufficiently many rounds ?

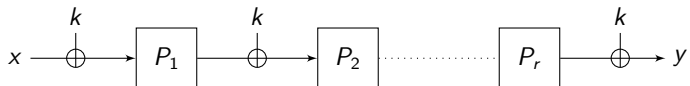
## Main Result

The single-key IEM with  $r = 12$  rounds is indifferentiable from an ideal cipher with  $n$ -bit blocks and  $n$ -bit keys

Also holds when using invertible permutations  $\gamma_i$  for the key derivation (no cryptographic assumption needed).

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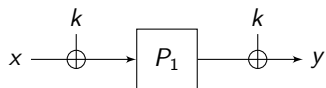
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## A simple attack for 1 round



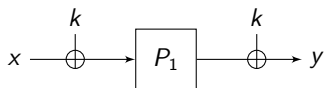
The distinguisher  $\mathcal{D}$  proceeds as follows:

- query  $P_1(a) = b$  for an arbitrary  $a$
- choose a random key  $k$  and define  $x = a \oplus k$
- query  $E(k, x) = y$  and check whether  $y = b \oplus k$  (\*)

Then:

- when  $\mathcal{D}$  interacts with a real EM cipher, (\*) always holds
- when  $\mathcal{D}$  interacts with  $(E, \mathcal{S}^E)$ , (\*) holds only with negligible probability since  $\mathcal{S}$  cannot guess  $k$  when answering the query  $P_1(a)$

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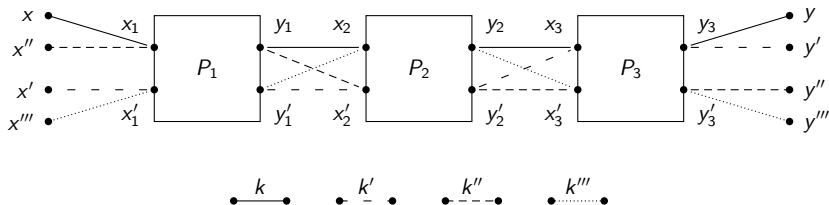
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# An attack for 3 rounds



One can (easily) find  $(x, x', x'', x''')$ ,  $(y, y', y'', y''')$  and  $(k, k', k'', k''')$  such that  $y = \text{EM}^{(P_1, P_2, P_3)}(k, x)$ , etc. and:

$$\begin{cases} k \oplus k' \oplus k'' \oplus k''' = 0 \\ x \oplus x' \oplus x'' \oplus x''' = 0 \\ y \oplus y' \oplus y'' \oplus y''' = 0 \end{cases} .$$

This can be showed to be hard for an ideal cipher.



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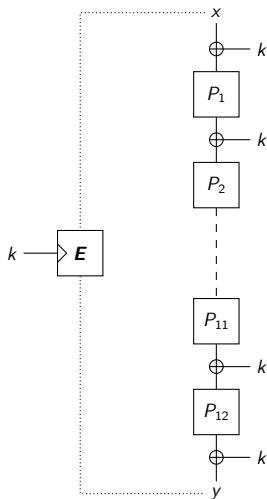
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# Simulation: general strategy

The simulator must return answers that are **coherent** with what the distinguisher can obtain from the ideal cipher  $E$ , i.e.:

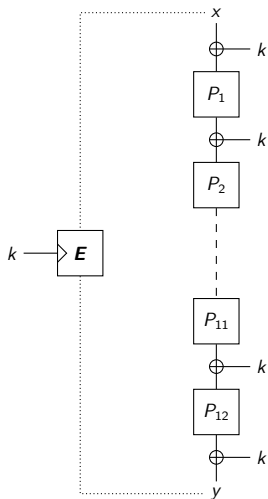
$$\text{EM}^{P_1, \dots, P_{12}}(k, x) = E(k, x)$$

For this, the simulator must **adapt** at least one permutation to “match” what is given by the ideal cipher



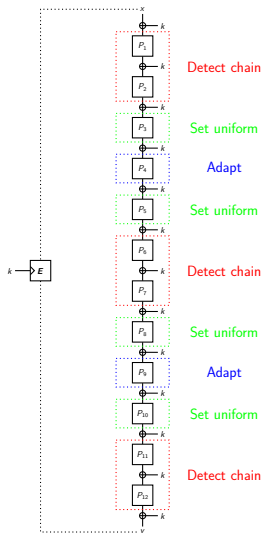
## Simulation: general strategy

- the simulator detects and completes “partial chains” = two adjacent queries  $P_i(x_i) = y_i$  and  $P_{i+1}(x_{i+1}) = y_{i+1}$
- for any partial chain the key is uniquely defined:  $k = y_i \oplus x_{i+1}$
- when a partial chain is detected, the simulator completes the missing permutation values randomly, except for one particular permutation which is “adapted” to match the ideal cipher



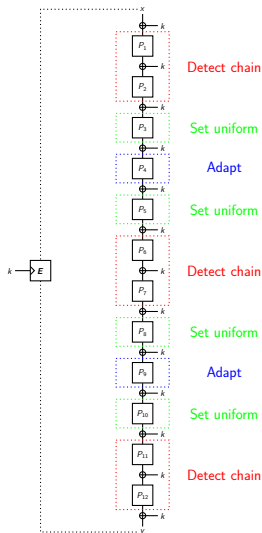
# How the simulator works

- the simulator only detects partial chains at very specific places:
  - external chains ( $P_1, P_2, P_{11}, P_{12}$ ) that matches the ideal cipher  $E$
  - central chains ( $P_6, P_7$ )
- an external chain can be created only if the distinguisher has made the corresponding query to  $E$ 
  - only  $q$  of them will be completed, which avoids an recursive blow-up of the simulator



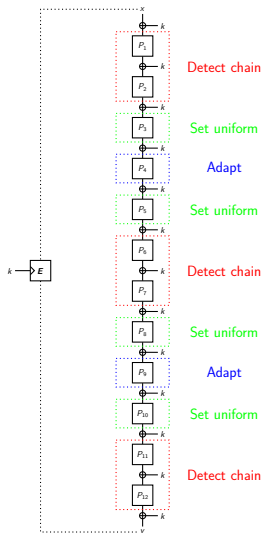
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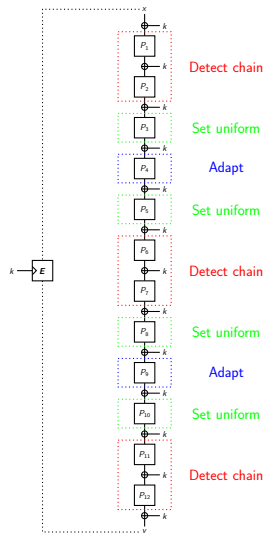
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# Open problems

The indifferentiability proof requires 12 rounds, but the best attack is only on 3 rounds.

## Conjecture

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Summary of results about the IEM cipher:

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- indifferentiability: the single-key IEM cipher with 12 rounds is indifferentiable from an ideal cipher with  $n$ -bit keys

Interpretation of the results:

- shows that the general strategy of building block ciphers from SPNs is sound and may even yield something close to an ideal cipher
- says little about concrete block ciphers: e.g. the permutations  $P_1, \dots, P_{10}$  of AES-128 are too simple
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The end...

Thanks for your attention!  
Comments or questions?

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