New Constructions and Applications of Trapdoor DDH Groups

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Introduction: CDH versus DDH

- group $\mathbb{G}$, element $G \in \mathbb{G}$ of large order
- CDH problem: given $X = G^x$ and $Y = G^y$, compute G^{xy}
- DDH problem: distinguish (G^x, G^y, G^{xy}) and (G^x, G^y, G^z)
- usual situations in cryptographic groups:
  1. CDH and DDH are both (presumably) hard
     $\rightarrow$ e.g. prime order subgroup of $\mathbb{Z}_p^*$
  2. CDH is (presumably) hard and DDH is universally easy
     $\rightarrow$ pairing groups

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Trapdoor DDH groups (TDDH groups):

- lies somewhere between cases 1 and 2:
  $\rightarrow$ CDH is hard, while DDH is hard unless one has some trapdoor τ
- introduced by Dent and Galbraith [DG06]
- very few constructions (hidden pairing construction by [DG06])
- very few applications:
  - simple identification scheme [DG06]
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Our contributions:

- we slightly refine the original definition of trapdoor DDH groups by [DG06]
- we introduce static trapdoor DDH groups
- we give new constructions of trapdoor DDH and static trapdoor DDH groups based on standard assumptions
- we show that (static) trapdoor DDH groups give very simple constructions of convertible undeniable signature schemes

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Outline

1. Definition of Trapdoor DDH Groups
2. New Constructions of TDDH and Static TDDH Groups
   - A TDDH group based on composite residuosity
   - A static TDDH group based on RSA
   - A static TDDH group based on factoring
3. Application to Convertible Undeniable Signatures

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TDDH group: definition

Trapdoor DDH group

$(\mathbb{G}, G, \tau) \leftarrow \textsc{GpGen}(1^k)$ is a trapdoor DDH group if:

1. the DDH problem is hard for $(\mathbb{G}, G)$ without the trapdoor τ
2. the CDH problem is hard even with the trapdoor τ
3. there is a distinguishing algorithm $\textsc{Solve}(X, Y, Z, \tau)$ which:
   - always accepts when (X, Y, Z) is a DDH tuple (completeness)
   - accepts with negligible probability for any adversarially generated $Z \leftarrow \mathcal{A}(X, Y)$ (soundness)

When $\textsc{Solve}$ always rejects on input a non-DDH tuple (X, Y, Z), we say that the TDDH group has perfect soundness.

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Dent and Galbraith originally proposed two TDDH group constructions:

1. disguised elliptic curve [Frey98]
   → broken by Morales [Mor08]

2. hidden pairing:
   - uses an elliptic curve E over the ring $\mathbb{Z}_N$, $N = p_1 p_2$
   - point $G \in E(\mathbb{Z}_N)$ of order $r_1 r_2$ where $r_1 | (p_1 + 1)$ and $r_2 | (p_2 + 1)$
   - the trapdoor is $\tau = (p_1, p_2, r_1, r_2)$
   - by the CRT, $(X, Y, Z) \in \langle G \rangle^3$ is a DDH tuple iff it reduces to a DDH tuple in $E(\mathbb{F}_{p_1})$ and $E(\mathbb{F}_{p_2})$
     → solve the DDH problem in $E(\mathbb{F}_{p_1})$ and $E(\mathbb{F}_{p_2})$ using a pairing
   - problem: no obvious way to hash into $\langle G \rangle$

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Static TDDH groups

Static TDDH group = more restricted variant of TDDH group
→ the trapdoor τ_x is dedicated to some fixed element X

Static trapdoor DDH group

$(\mathbb{G}, G, \tau) \leftarrow \textsc{GpGen}(1^k)$ is a static TDDH group if there is a randomized algorithm $(X, \tau_x) \leftarrow \textsc{Sample}(\tau)$ taking the master trapdoor τ as input such that:

1. the DDH problem is hard for $(\mathbb{G}, G)$ without the trapdoor τ
2. the static CDH problem for (G, X) is hard even given τ_x
3. there is a distinguishing algorithm $\textsc{Solve}(X, Y, Z, \tau_x)$ which distinguishes DDH tuples from non-DDH tuples

Remark: in a static trapdoor DDH group, the Strong Diffie-Hellman problem (*i.e.* solving the CDH problem given a static DDH oracle) is hard

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A TDDH group based on composite residuosity [BCP03]

- $N = pq$, with p, q safe primes
- $\mathbb{G} = \mathbb{QR}_{N^2}$ is the group of quadratic residues mod N^2
- G generator of $\mathbb{G}$

Partial discrete log (Paillier [Pai99])

Given the factorization of N, it is possible to compute efficiently the partial discrete log defined as:

$$\mathtt{PDlog}_G(X) := \mathtt{Dlog}_G(X) \bmod N \ .$$

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- trapdoor $\tau = (p, q)$

Solving the DDH problem in $(\mathbb{G}, G)$ using trapdoor $\tau = (p, q)$:

- input $(X, Y, Z) \in \mathbb{G}^3$
- compute $x' = \mathtt{PDlog}_G(X)$, $y' = \mathtt{PDlog}_G(Y)$, $z' = \mathtt{PDlog}_G(Z)$
- check whether $x'y' = z' \bmod N$

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- The soundness property relies on the following problem:

Partial CDH problem

Given N and G generator of $\mathbb{G} = \mathbb{QR}_{N^2}$, and $X, Y \leftarrow_\$ \mathbb{G}$, output Z such that $\mathtt{PDlog}_G(Z) = \mathtt{PDlog}_G(X) \times \mathtt{PDlog}_G(Y) \bmod N$.

- Issue: this TDDH group does not have perfect soundness
  The $\textsc{Solve}$ algorithm accepts even for a non-DDH tuple (X, Y, Z) such that $\mathtt{PDlog}_G(Z) = \mathtt{PDlog}_G(X) \times \mathtt{PDlog}_G(Y) \bmod N$.
- Given a DDH tuple (X, Y, Z), anyone can compute $Z' = ZU^N$, and (X, Y, Z') is a non-DDH tuple which fools the $\textsc{Solve}$ algorithm
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A static TDDH group based on RSA

- $\textsc{GpGen}(1^k)$:
  - $N = pq$, with p, q safe primes
  - $\mathbb{G} = \mathbb{J}_N$ is the subgroup of $\mathbb{Z}_N^*$ of elements with Jacobi symbol 1
  - G generator of $\mathbb{G}$
  - master trapdoor $\tau = (p, q)$
- sampling a group element and the corresponding trapdoor:
  - draw $x \leftarrow_\$ \{1, \ldots, |\mathbb{J}_N|\}$, let $X = G^x$
  - the trapdoor is $\tau_x = 1/x \bmod \mathrm{ord}(\mathbb{J}_N)$
- solving the DDH problem for $(X, Y, Z) \in \mathbb{G}^3$:
  $\rightarrow$ check whether $Z^{\tau_x} = Y$ (satisfied iff $Z = Y^x$)
- Theorem: Under the DDH assumption and the RSA assumption, this is a static TDDH group with perfect soundness
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    $\mathbb{J}_N^+ \simeq \mathbb{J}_N / \{+1, -1\}$ (group of *signed quadratic residues* [HK09])
  - generator G of $\mathbb{G}$
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- Theorem: Under the DDH assumption and the factoring assumption, this group is a static TDDH group with perfect soundness
- NB: implies that Strong DH is hard for $\mathbb{J}_N^+$ under the factoring assumption [HK09]

A static TDDH group based on factoring

- $\textsc{GpGen}(1^k)$:
  - $N = pq$, with p, q safe primes
  - $\mathbb{G} = \mathbb{J}_N^+ = \mathbb{J}_N \cap [1, (N-1)/2]$, group operation: $a * b := |a \cdot b \bmod N|$
    $\mathbb{J}_N^+ \simeq \mathbb{J}_N / \{+1, -1\}$ (group of *signed quadratic residues* [HK09])
  - generator G of $\mathbb{G}$
  - master trapdoor $\tau = (p, q)$
- sampling a group element and the corresponding trapdoor:
  - draw $x \leftarrow_\$ \{1, \ldots, |\mathbb{J}_N^+|\}$, let $X = G^x$
  - the trapdoor is $\tau_x = 2x \pm m$ with $m = \mathrm{ord}(\mathbb{J}_N^+)$
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Hashing into groups

For both previous cases, it is possible to securely hash into the underlying group $\mathbb{G}$.

Given $H : \{0,1\}^* \rightarrow \mathbb{Z}_N$, let a be an integer with $\left(\frac{a}{N}\right) = -1$

- for $\mathbb{G} = \mathbb{J}_N$, define

$$H'(x) = \begin{cases} H(x) & \text{if } \left(\frac{H(x)}{N}\right) = 1 \\ a \cdot H(x) \bmod N & \text{if } \left(\frac{H(x)}{N}\right) = -1 \end{cases}$$

- for $\mathbb{G} = \mathbb{J}_N^+$, define

$$H'(x) = \begin{cases} |H(x)| & \text{if } \left(\frac{H(x)}{N}\right) = 1 \\ |a \cdot H(x) \bmod N| & \text{if } \left(\frac{H(x)}{N}\right) = -1 \end{cases}$$

Outline

Definition of a CUS scheme

Undeniable signature = signature that cannot be verified without the cooperation of the signer

Convertible Undeniable Signature Scheme:

- $\texttt{KeyGen}(1^k)$: outputs a public/secret key pair $(\texttt{pk}, \texttt{sk})$ for the signer.
- $\texttt{USign}(\texttt{pk}, \texttt{sk}, m)$: outputs an undeniable signature σ for message m.
- $\Pi_{\text{con}} = (\mathcal{P}_{\text{con}}, \mathcal{V}_{\text{con}})$: confirmation protocol for a valid signature σ
- $\Pi_{\text{dis}} = (\mathcal{P}_{\text{dis}}, \mathcal{V}_{\text{dis}})$: disavowal protocol for an invalid signature σ'
- $\texttt{UConvert}(\texttt{pk}, \texttt{sk})$: outputs a universal receipt ρ_u enabling to universally verify signatures created under $(\texttt{pk}, \texttt{sk})$.
- $\texttt{UVer}(\texttt{pk}, \rho_u, m, \sigma)$: signature verification algorithm using the universal receipt ρ_u

The Chaum-van Antwerpen scheme [CvA89]

Parameters:

- a group $\mathbb{G}$ and a gen. G such that the DDH problem is hard
- a hash function $H : \{0,1\}^* \rightarrow \mathbb{G}$

CvA undeniable signature scheme

- Key generation: $\mathtt{sk} := x \leftarrow_\$ \{1, \ldots, |\langle G \rangle|\}$, $\mathtt{pk} := X := G^x$
- Signing a message m: compute $M = H(m) \in \mathbb{G}$, and $S = M^x$
- Confirming a sig. S for m: prove that $(X, H(M), S)$ is a DDH tuple
  $\rightarrow$ Chaum-Pedersen proof of equality of DL [CP92]
- Denying a sig. S' for m: prove that $(X, H(M), S')$ is a non-DDH tuple
  $\rightarrow$ Camenish-Shoup proof of inequality of DL [CS03]

Note: using a pairing group where DDH is easy yields the Boneh-Lynn-Shacham signature scheme [BLS04]

The CvA scheme with a static TDDH group

Using the CvA scheme with a (static) TDDH group gives new properties.

$\rightarrow$ New $\texttt{KeyGen}$: $(\mathbb{G}, G, \tau) \leftarrow \textsc{GpGen}(1^k)$, $(X, \tau_X) \leftarrow \textsc{Sample}(\tau)$

- signer public key: $\mathrm{pk} = X = G^x$
- signer secret key: $\mathrm{sk} = (x, \tau_X)$, where τ_X is the trapdoor for solving the static DDH problem for X

The signer now can use the trapdoor τ_X as follows:

- delegated verification: disclose the trapdoor τ_X to the delegated verifier DV
  $\rightarrow$ DV can confirm/disavow signatures using witness τ_X
- universal convertibility: simply make the trapdoor τ_X public
  $\rightarrow$ anyone can verify signatures S using $\textsc{Solve}(X, H(m), S, \tau_X)$

Caveat: requires perfect soundness

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Security properties:

- unforgeability under CMA attacks:
  $\rightarrow$ relies on hardness of the CDH problem (even given τ_x)
- invisibility under CMA attacks (impossibility to distinguish a valid signature from an random one):
  $\rightarrow$ relies on hardness of the DDH problem (without τ_x)

Instantiations

The Chaum-van Antwerpen scheme can be instantiated with the two proposed static TDDH groups:

- RSA-based static TDDH group $\mathbb{J}_N$:
  $\rightarrow$ scheme similar to the one by Gennaro, Rabin, and Krawczyk [GRK00]
- factoring-based static TDDH group $\mathbb{J}_N^+$:
  $\rightarrow$ scheme similar to the one by Galbraith and Mao [GM03]

Key generation must be done with care. One needs to certify that:

- $\mathbb{Z}_N^*$ has no small order subgroup
- G is a generator of the specified group

$\rightarrow$ demand that the signer proves in ZK that N is a product of safe primes

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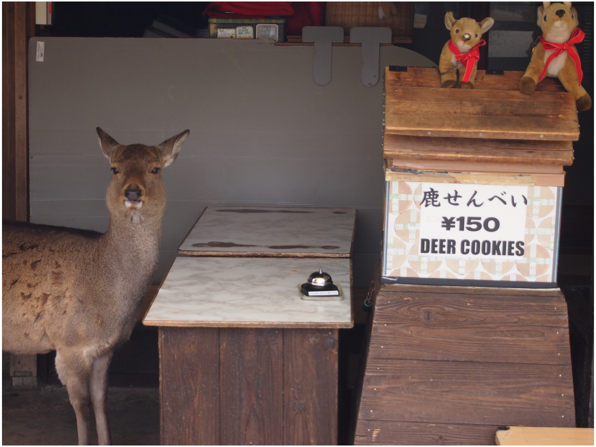
Conclusion

Open problems:

- build a TDDH group with perfect soundness and a way to securely hash into it
- build a TDDH group with prime order
- other applications of TDDH groups?
  $\rightarrow$ suggested by a PKC reviewer:
  generic construction of extractable hash proof system [Wee10]
  $\Rightarrow$ CCA-secure KEM

Thanks for your attention! Comments or questions?



Damn! Where's my wallet?