Efficiency and Privacy Improvements for Bitcoin with Schnorr Signatures

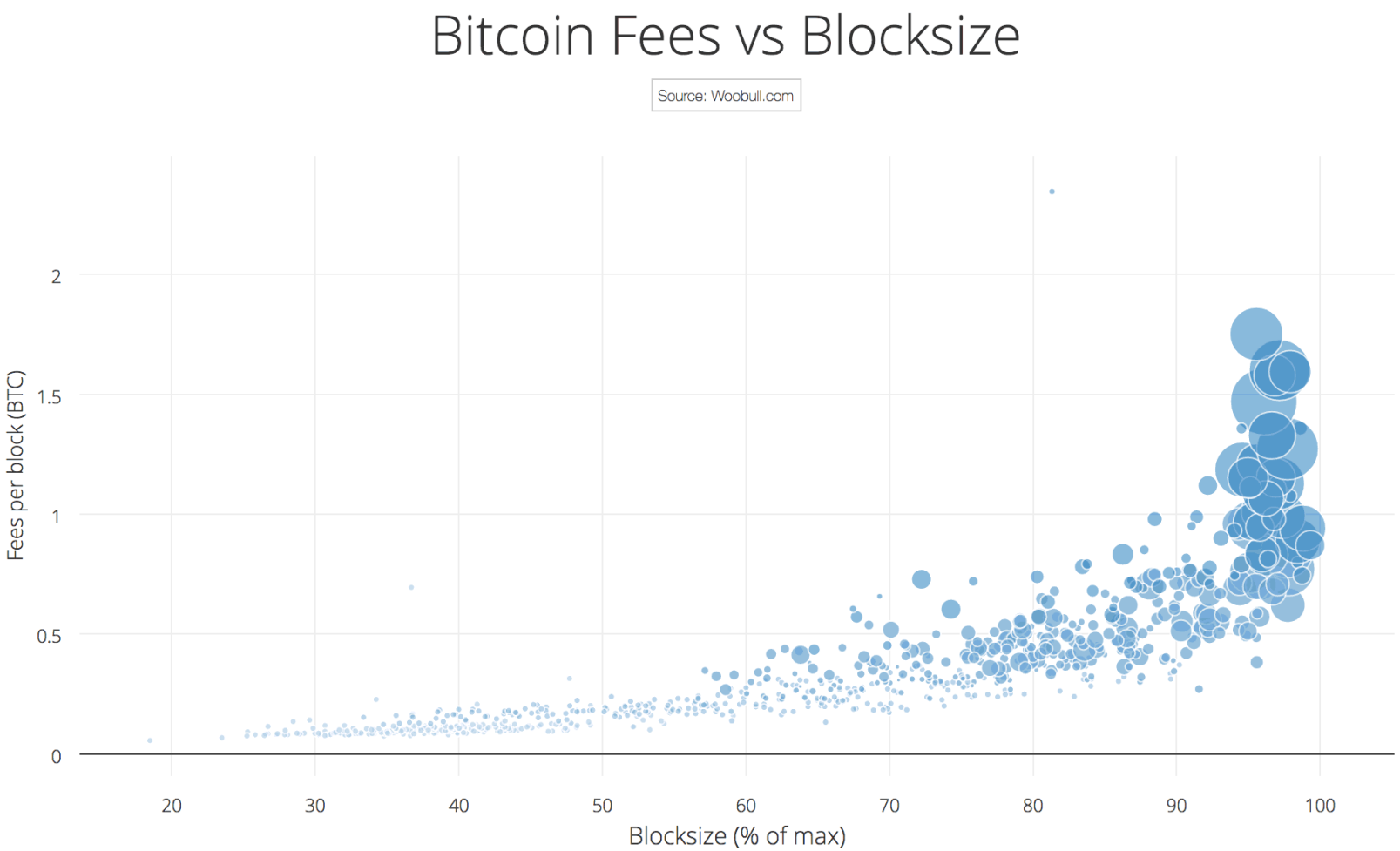
Yannick Seurin

(Based on joint work with
G. Maxwell, A. Poelstra, and P. Wuille)

Agence nationale de la sécurité des systèmes d'information

September 20, 2018 — "BlockSem" Seminar

Motivation: scalability problems

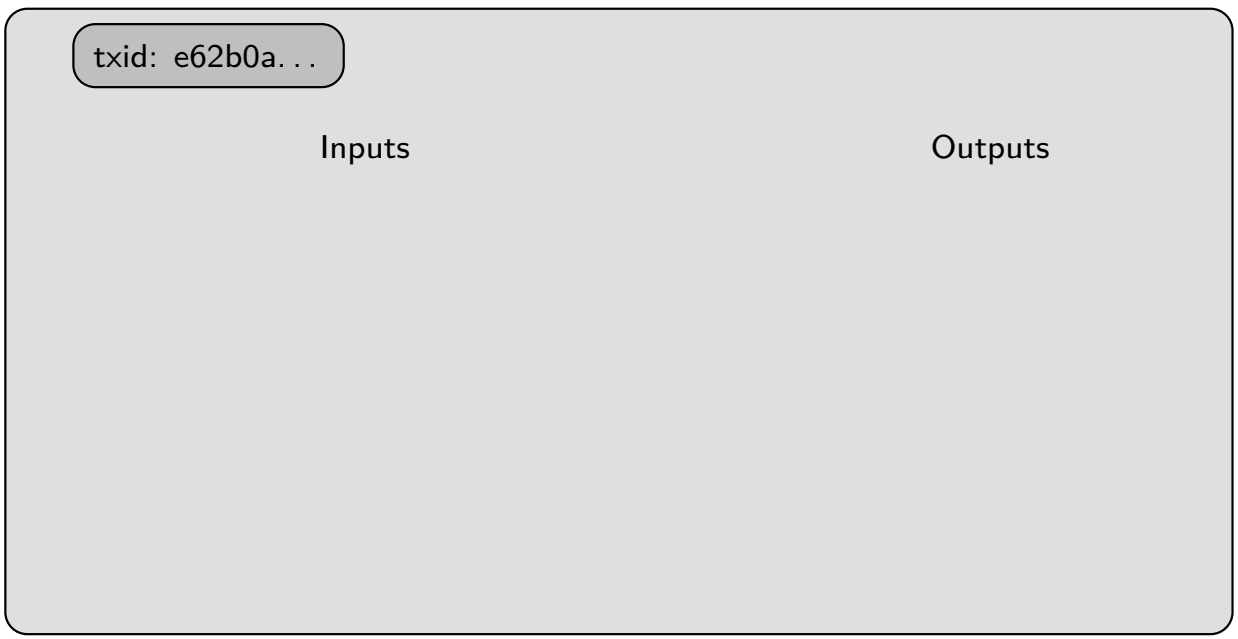


Miners fee per block vs block size (% of maximum), samples grouped by day. Bubble size denotes mempool size (for records Apr 2016 onwards).

Bitcoin transactions

A Bitcoin transaction spends inputs and creates outputs:

- an input consists of a reference to an output of a previous transaction and a signature authorizing spending of this output
- an output consists of an amount and a public key



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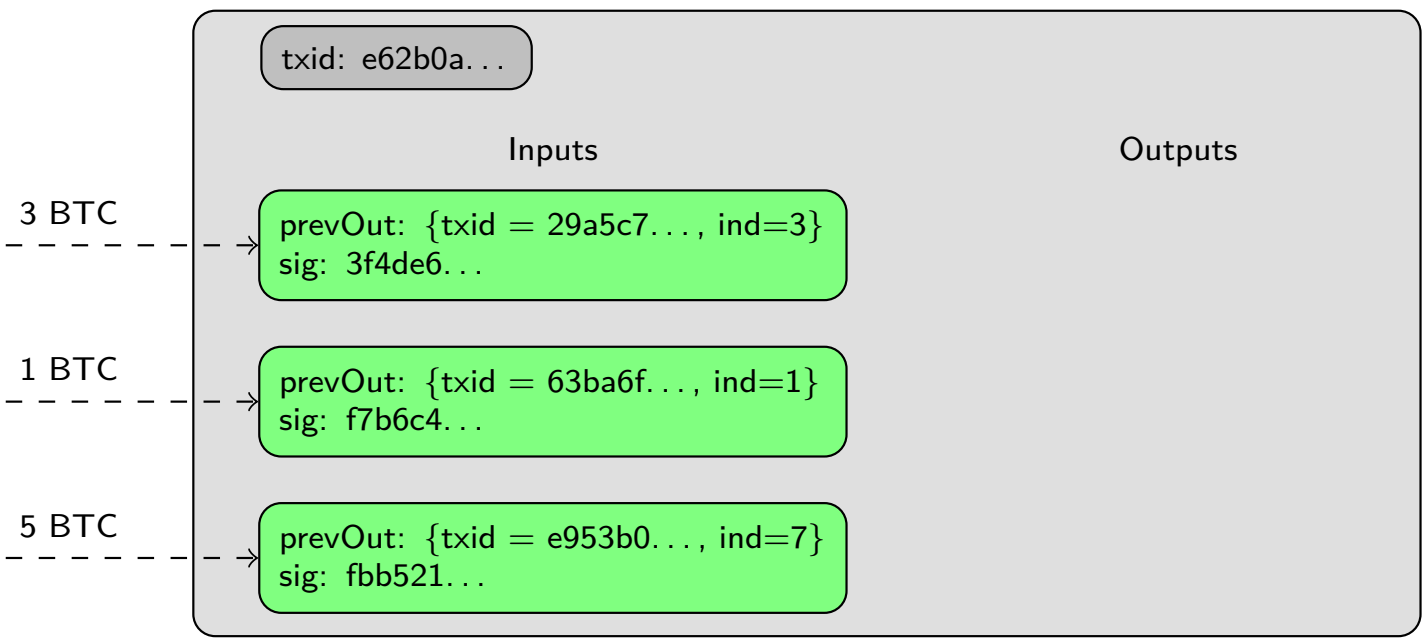

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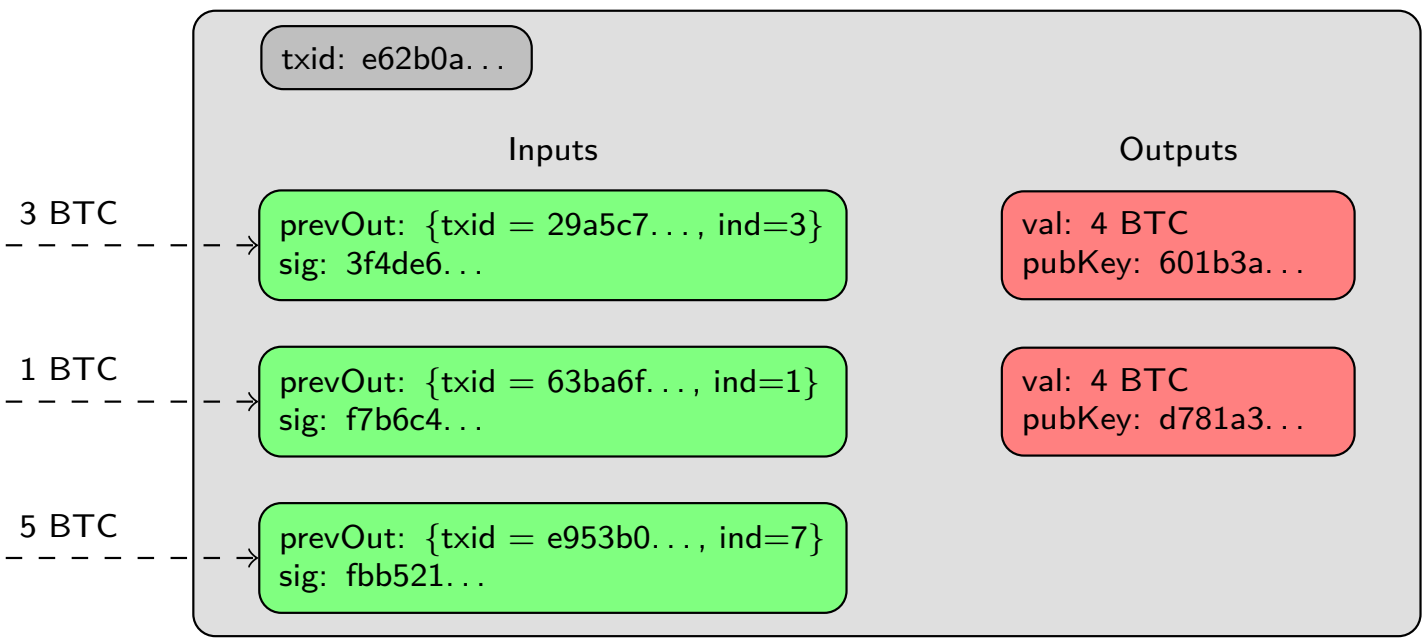

Signatures in Bitcoin

- to spend an output, users must provide a signature proving ownership
- spending a P2PKH output requires one signature
- spending a m-of-n multisig output (P2MS or P2SH) requires m signatures (and n public keys)
- signature data $\Rightarrow$ transaction data $\Rightarrow$ transaction fees (BTC/byte)
- typical size of an ECDSA signature over secp256k1 (two 32-bytes integers + 6 bytes DER encoding) = 72 bytes
- 300 000 000 transactions in the blockchain, ~ 2 inputs/tx
  $\Rightarrow$ at least 54 GB of signature data (28% blockchain size)
- could we use less signatures and less public keys without harming security?

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MuSig: Schnorr-based multi-signatures

Simple Schnorr Multi-Signatures with Applications to Bitcoin

Gregory Maxwell, Andrew Poelstra[1], Yannick Seurin[2], and Pieter Wuille[1]

[1] Blockstream
[2] ANSSI, Paris, France
greg@xiph.org,
{apoelstra, pwuille}@blockstream.com,
yannick.seurin@m4x.org

January 15, 2018

Abstract. We describe a new Schnorr-based multi-signature scheme (i.e., a protocol which allows a group of signers to produce a short, joint signature on a common message), provably secure in the plain public-key model (meaning that signers are only required to have a public key, but do not have to prove knowledge of the private key corresponding to their public key to some certification authority or to other signers before engaging the protocol), which improves over the state-of-art scheme of Bellare and Neven (ACM-CCS 2006) and its variants by Bagherzandi *et al.* (ACM-CCS 2008) and Ma *et al.* (Des. Codes Cryptogr., 2010) in two respects: (i) it is simple and efficient, having only two rounds of communication instead of three for the Bellare-Neven scheme and the same key and signature size as standard Schnorr signatures; (ii) it allows *key aggregation*, which informally means that the joint signature can

https://eprint.iacr.org/2018/068.pdf

Outline

Signature Schemes: Schnorr versus ECDSA

Signature and Key Aggregation

Other Applications

Conclusion

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History of discrete log-based signature schemes

- 1984: ElGamal signatures
- 1985: Elliptic Curve Cryptography proposed by Koblitz and Miller
- 1989: Schnorr signatures, U.S. Patent 4,995,082
- 1991: DSA (*Digital Signature Algorithm*) proposed by NIST
- 1992: ECDSA (*Elliptic Curve DSA*) proposed by Vanstone
- 1993: DSA standardized by NIST as FIPS 186
- 2000: ECDSA included in FIPS 186-2
- 2008: Schnorr's patent expires
- 2009: Bitcoin is launched

C.P. Schnorr

Signature scheme: definition

A signature scheme consists of three algorithms:

1. key generation algorithm `Gen`:
   - returns a public/secret key pair (pk, sk)
2. signature algorithm `Sign`:
   - takes as input a secret key sk and a message m
   - returns a signature σ
3. verification algorithm `Ver`:
   - takes as input a public key pk, a message m, and a signature σ
   - returns 1 if the signature is valid and 0 otherwise

Correctness property:

$$\forall (pk, sk) \leftarrow \texttt{Gen},\ \forall m,\ \texttt{Ver}(pk, m, \texttt{Sign}(sk, m)) = 1$$

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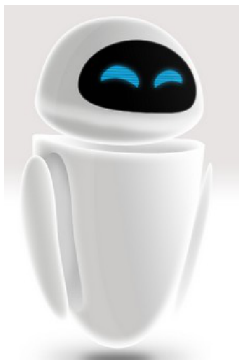
Signature scheme: security

sk_A

pk_A



- "gold" security notion: Existential Unforgeability against Chosen Message Attacks (EUF-CMA)
- strong-EUF-CMA: $(m^*, \sigma^*) \neq (m_1, \sigma_1), \ldots, (m_q, \sigma_q)$
- strong-EUF-CMA $\Leftrightarrow$ EUF-CMA + non-malleability

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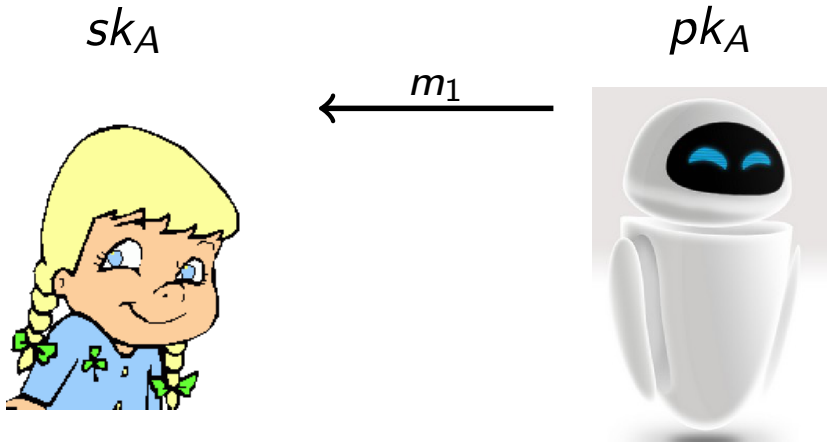

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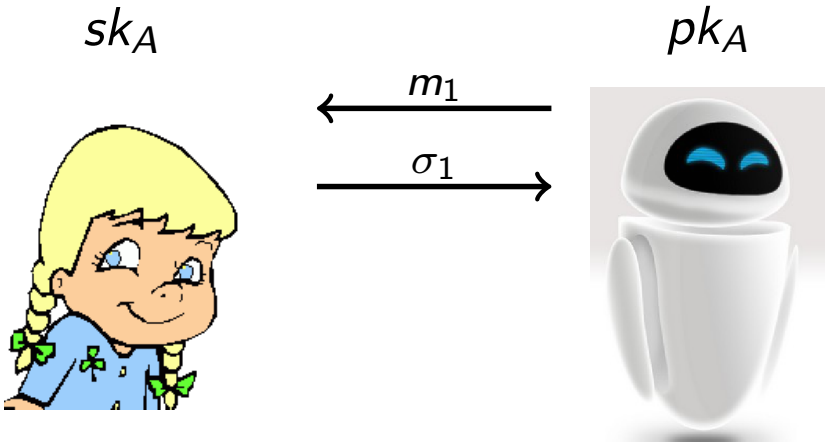

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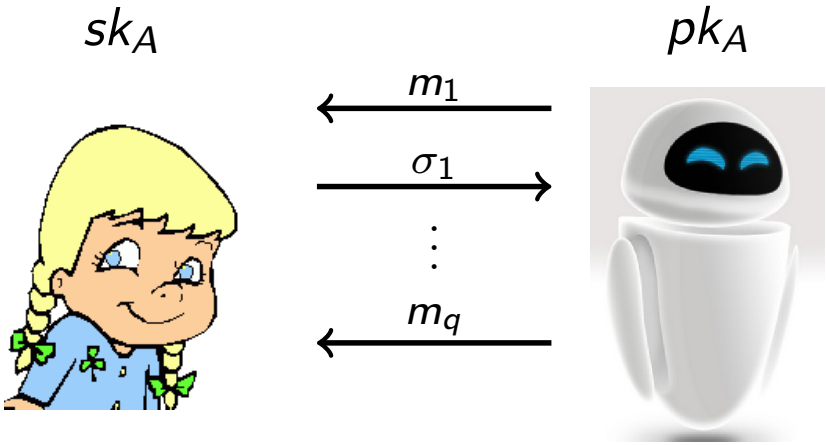

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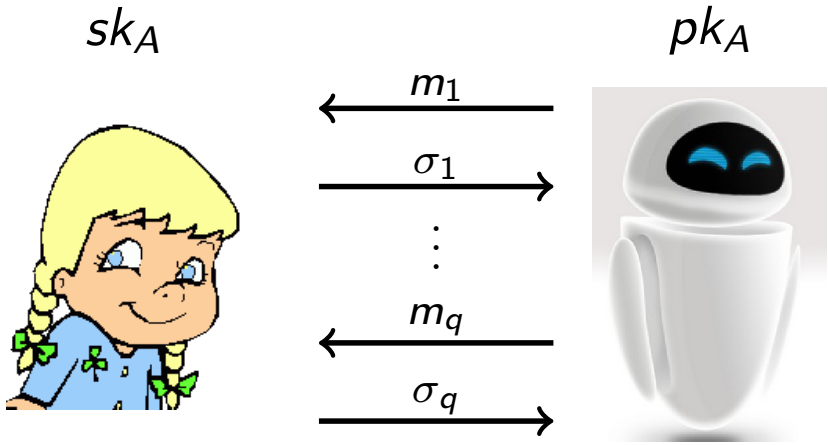

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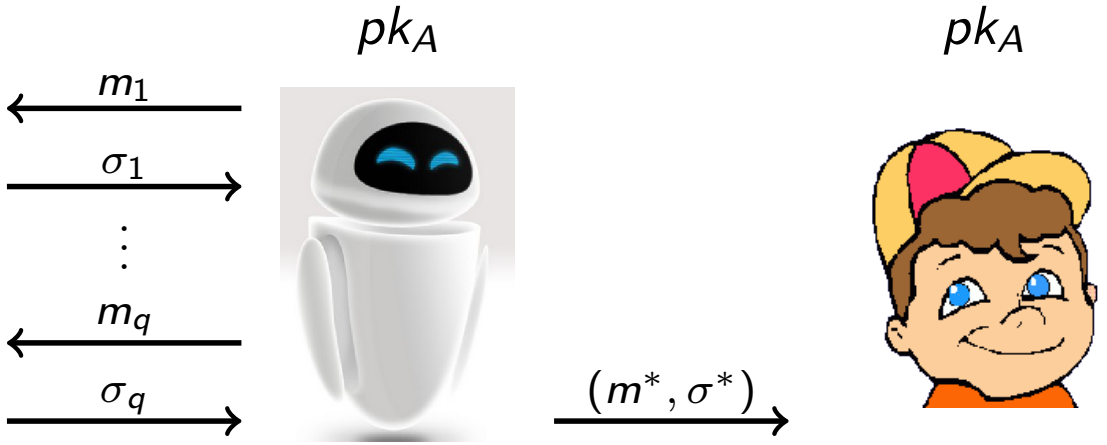

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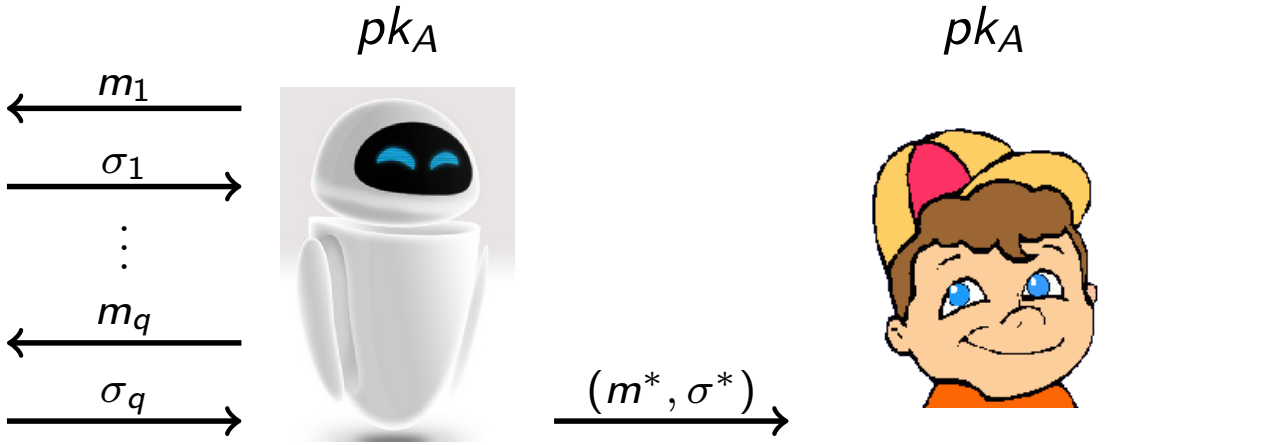

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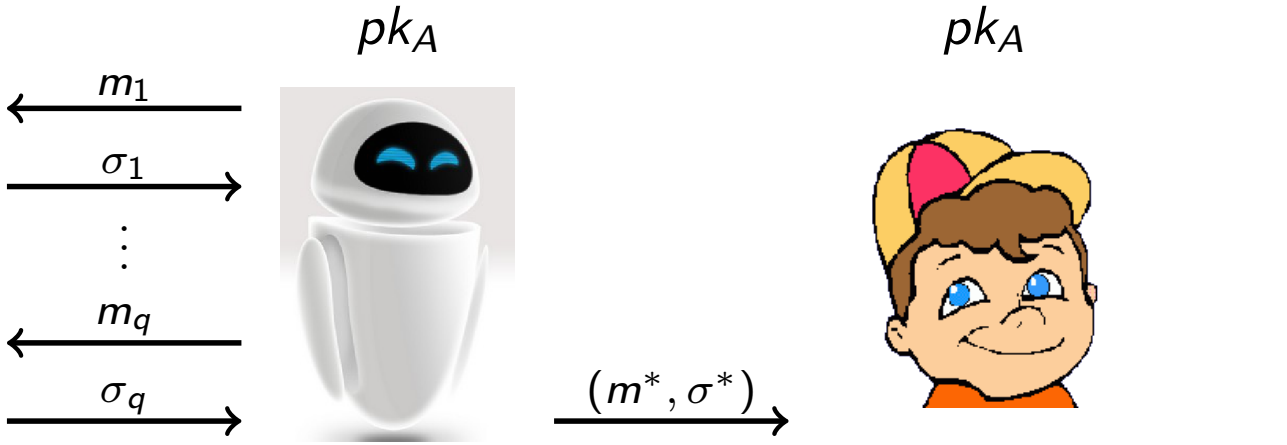

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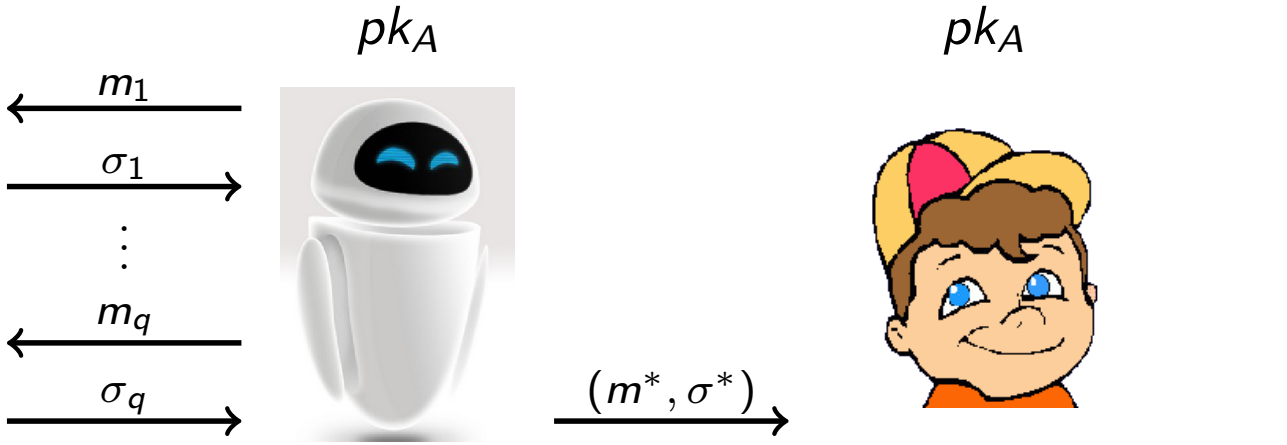

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Mathematical background

Cyclic group and generator

Let $\mathbb{G}$ be an abelian group of order p. An element $G \in \mathbb{G}$ is called a *generator* if

$$\langle G \rangle \stackrel{\text{def}}{=} \{0G, 1G, 2G, \ldots\} = \mathbb{G}.$$

If G is a generator, then for any $X \in \mathbb{G}$, there exists a unique $x \in \{0, \ldots, p-1\}$ such that $X = xG$.

Discrete logarithm problem

Given $X \in \mathbb{G}$, find $x \in \{0, \ldots, p-1\}$ such that $X = xG$.

NB: with multiplicative notation, $xG \sim G^x$

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Schnorr authentication protocol [Sch89, Sch91]

Public parameters: a cyclic group $\mathbb{G}$ of prime order p, a generator G of $\mathbb{G}$

$$\begin{cases} sk_{\text{Alice}} = x \leftarrow_{\$} \mathbb{Z}_p \\ pk_{\text{Alice}} = xG = X \end{cases}$$

$pk_{\text{Alice}} = X$

$r \leftarrow_{\$} \mathbb{Z}_p,\ R = rG$ —— R ⟶

⟵ c —— $c \leftarrow_{\$} \mathbb{Z}_p$

$s = r + cx \bmod p$ —— s ⟶ Check $sG \stackrel{?}{=} R + cX$

Schnorr's protocol is a "proof of knowledge"

Theorem

Schnorr's protocol is secure against impersonation under the discrete logarithm assumption.

Proof.

- assume there exists an attacker $\mathcal{A}$ which is able to authenticate with good probability
- we run $\mathcal{A}$ on public key X: it sends $R = rG$, we answer with c_1, and $\mathcal{A}$ returns the correct answer $s_1 = r + c_1 x \bmod p$
- we rewind $\mathcal{A}$ and run it again: it sends $R = rG$, we answer with $c_2 \neq c_1$, and $\mathcal{A}$ returns the correct answer $s_2 = r + c_2 x \bmod p$
- we compute $x = (s_1 - s_2)(c_1 - c_2)^{-1} \bmod p$ □

The Fiat-Shamir transform [FS86]

- it is easy to obtain a valid transcript (R, c, s) without knowledge of the secret key x by computing "backwards":
  - choose $s \leftarrow_{\$} \mathbb{Z}_p$
  - choose $c \leftarrow_{\$} \mathbb{Z}_p$
  - compute $R = sG - cX$
- what convinces Bob is that he knows that c was chosen after R was committed by Alice
- how could we make the protocol non-interactive?
- answer: replace the verifier (Bob) by a hash function H
- Alice computes the challenge by herself as $c = H(X, R)$
- assuming H "behaves randomly", this can be proved secure (random oracle model)

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  - a cyclic group $\mathbb{G}$ of prime order p and a generator G
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- key generation:
  - secret key $x \leftarrow_{\$} \mathbb{Z}_p$
  - public key $X = xG$
- signature: on input m and x,
  - draw $r \leftarrow_{\$} \mathbb{Z}_p$ and compute $R = rG$
  - compute $c = H(X, R, m)$ and $s = r + cx \bmod p$
  - output $\sigma = (R, s)$
- verification: on input X, m and $\sigma = (R, s)$,
  - compute $c = H(X, R, m)$ and check $sG \stackrel{?}{=} R + cX$
- alternative:
  - signature $\sigma = (c, s)$
  - verification: compute $R = sG - cX$ and check $H(X, R, m) \stackrel{?}{=} c$

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- alternative:
  - signature $\sigma = (c, s)$
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Schnorr signatures [Sch89, Sch91]

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  - compute $c = f(R)$ and $s = r^{-1}(H(m) + cx) \bmod p$
  - output $\sigma = (c, s)$
- verification: on input X, m and $\sigma = (c, s)$,
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(EC)DSA signatures

DSA and ECDSA are instantiations of the "generic" DSA scheme:

- for DSA:
  - $\mathbb{G}$ = cyclic subgroup of prime order p of $\mathbb{Z}_q^*$ for some large prime q ($|q| \geq 3072$ bits)
  - conversion function: $f(X) = X \bmod p$
- for ECDSA:
  - $\mathbb{G}$ = cyclic subgroup of prime order p of an elliptic curve group over some finite field ($\mathbb{F}_q$ for q prime or $q = 2^n$)
  - for q prime, group elements are pairs of integers $(x, y) \in \mathbb{F}_q^2$ satisfying the curve equation $E : y^2 = x^3 + ax + b$
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  - Bitcoin uses curve **secp256k1** [SEC10] (not a NIST curve!) (Standards for Efficient Cryptography, Koblitz curve over prime field $\mathbb{F}_q$ where $q = 2^{256} - 2^{32} - 977$, $a = 0$, $b = 7$)
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- ECDSA is not strongly EUF-CMA
- given a valid signature (c, s) for message m, it is possible to "maul" a different signature which is also valid, namely $(c, -s \bmod p)$
- verification equations:

(c, s)	$(c, -s)$
$u = H(m)s^{-1} \bmod p$	$u' = -H(m)s^{-1} \bmod p = -u$
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Schnorr versus ECDSA

- Schnorr security:
  - Schnorr signatures have a security proof under the Discrete Logarithm assumption in the Random Oracle Model for H [PS96]
  - no known attacks against Schnorr based on H collisions
- ECDSA security:
  - security analysis of (EC)DSA is much more brittle [Bro05] (uses generic group model, proves non-malleability!)
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Schnorr versus ECDSA: Summary

	Schnorr: $\sigma = (R, s)$	ECDSA: $\sigma = (c, s)$
`Ver`	$sG \stackrel{?}{=} R + H(R, m)X$	$f\left(H(m)s^{-1}G + cs^{-1}X\right) \stackrel{?}{=} c$
Fiat-Shamir	✓	×
sec. proof	✓	×
H	2nd preimage	collision
non-mall.	✓	×
batch ver.	✓	×

Outline

Signature Schemes: Schnorr versus ECDSA

Signature and Key Aggregation

Other Applications

Conclusion

What is a multi-signature protocol?

- assume n signers with public keys $\{pk_1, \ldots, pk_n\}$ want to sign the same message (e.g., spending from an n-of-n multisig address)
- trivial solution: compute one signature for each pk_i and output $\Sigma = (\sigma_1, \ldots, \sigma_n)$
- problem: the length of Σ grows linearly with the number of signers. Can we do better? (Ideally, the size of the "multi-signature" should be independent from the number of signers)
- well-studied problem in cryptography originally tackled in [IN83]
- hard to achieve for ECDSA due to its complex algebraic structure (modular division)

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"Naive" Schnorr multi-signatures

- Alice's public key $X_1 = x_1 G$, Bob's public key $X_2 = x_2 G$
- signature protocol:
  - Alice draws $R_1 = r_1 G$, Bob draws $R_2 = r_2 G$
  - they both compute $R = R_1 + R_2 = (r_1 + r_2)G$
  - they both compute $c = H(X_1, X_2, R, m)$
  - Alice computes $s_1 = r_1 + c x_1 \bmod p$
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- assume that signers can claim whatever public key they want (plain public key model)
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- variant of BN where the challenge for the i-th signer is

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$$c_i = \underbrace{H_0(L, X_i)}_{a_i} \underbrace{H_1(\widetilde{X}, R, m)}_{c} \quad \text{where} \quad \widetilde{X} = \sum_{i=1}^{n} H_0(L, X_i) X_i$$

- partial signature $s_i = r_i + c a_i x_i \bmod p$, $s = \sum_{i=1}^{n} s_i \bmod p$
- $\widetilde{X}$ is called the aggregated key
- verification identical to "normal" signature with public key $\widetilde{X}$:

$$sG = R + \sum_{i=1}^{n} c_i X_i = R + \underbrace{H_1(\widetilde{X}, R, m)}_{c} \underbrace{\sum_{i=1}^{n} H_0(L, X_i) X_i}_{\widetilde{X}}$$

- variant with $\widetilde{X} = \sum_{i=1}^{n} H_0(X_i) X_i$ is insecure (Wagner's algorithm)

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Application 1: replacing OP_CHECKMULTISIG

- using MuSig, n-of-n multisig outputs can be replaced by standard P2PKH output for the aggregated key $\widetilde{X}$
- this improves privacy
  - individual public keys are never revealed
  - the resulting output is indistinguishable from a standard P2PKH output
- for "threshold" m-of-n multisigs with $m < n$:
  - build a Merkle tree where leaves are all $\binom{n}{m}$ possible aggregated keys and only put the root in the `ScriptPubKey`
  - to spend, give a Merkle proof of membership of some $\widetilde{X}$ and a signature valid for $\widetilde{X}$

Application 2: cross-input signature aggregation

- transaction with multiple inputs: each key signs a different message
- $\Rightarrow$ Interactive Aggregate Signature (IAS) scheme
- BN proposed to use a multi-signature scheme with message $M = m_1 \| m_2 \| \ldots \| m_n$ (generic conversion MS $\rightarrow$ IAS)
- insecure in the plain public key model (credit: R. O'Connor):
  - Alice has two outputs O_1 and O_2 (same pub. key $X_a = x_a G$)
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- a secure IAS scheme requires "breaking the symmetry" for signers with the same public key
- solution: modify BN to compute the challenge for i-th signer as

$$c_i = H(\{(X_1, m_1), \ldots, (X_n, m_n)\}, R, i)$$

- the previous attack does not work since Alice computes

$$c_1 = H(\{(X_a, m_1), (X_a, m_2)\}, R, 1)$$

whereas Bob must use

$$c_2 = H(\{(X_a, m_1), (X_a, m_2)\}, R, 2)$$

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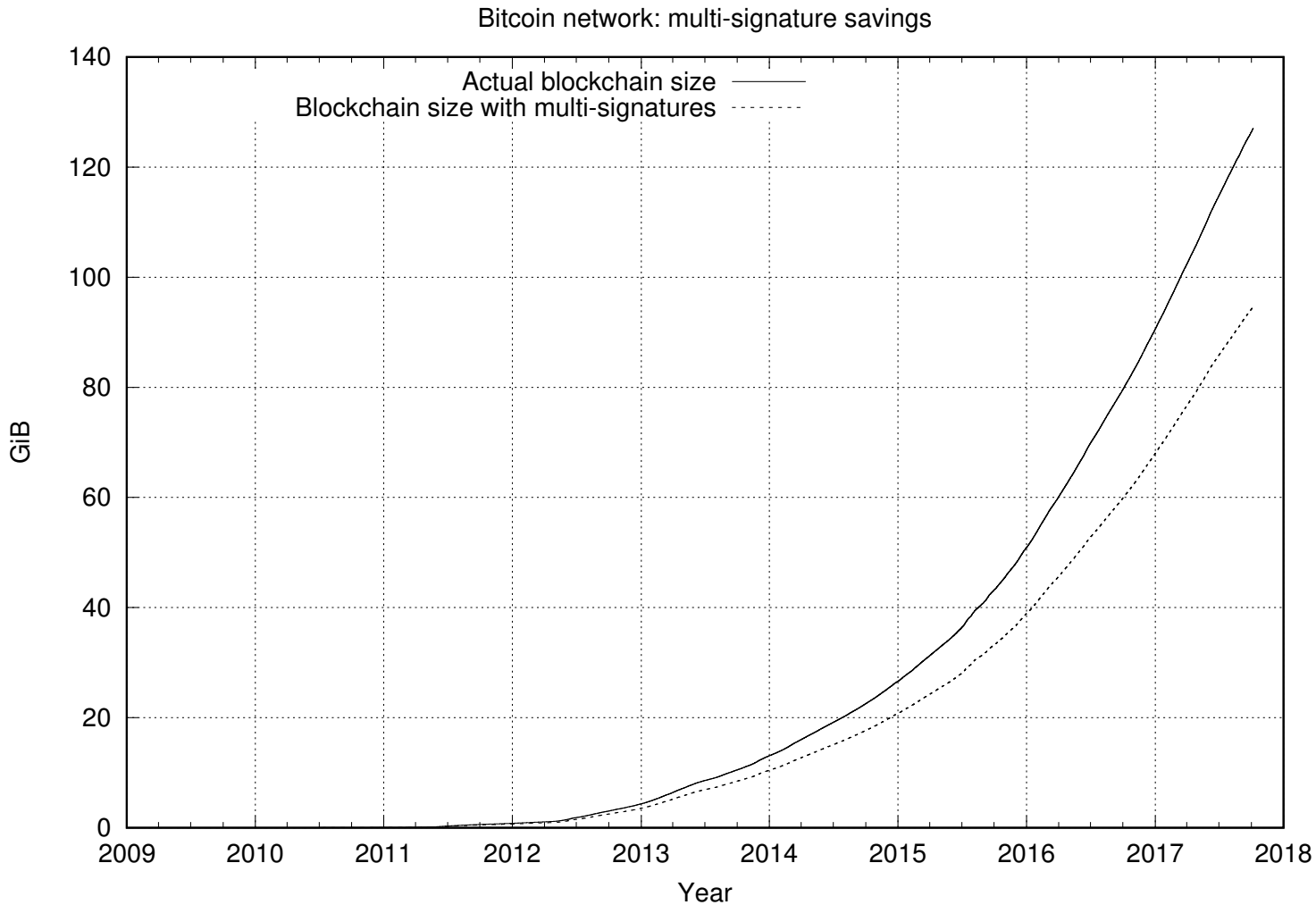
Benefits: Space savings



Benefits: UTXO set consolidation

- actors handling a large number of transactions can end up with a large number of "dust" UTXOs (e.g. exchanges)

LaurentMT @LaurentMT · 21 déc. 2017
For example, this entity (oxt.me/entity/tiid/48...) is a wallet controlled by Coinbase. To date, it owns around 203 BTC split in 1,464,545 utxos !
With BTC at $15.8k, it means $3.2M with an average utxo value of 2.2$.
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- they become impossible to spend when fees are too high
- cross-input signature aggregation allows to merge them into a single UTXO with a single signature rather than one signature per input $\Rightarrow$ lower transaction fees

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Outline

Signature Schemes: Schnorr versus ECDSA

Signature and Key Aggregation

Other Applications

Conclusion

Taproot (G. Maxwell)

- conditions for spending an output often of the form

$$\underbrace{(n \text{ parties agree to sign})}_{n\text{-of-}n \text{ multisig}} \text{ OR } \underbrace{(\text{some more complex conditions})}_{\text{script } S}$$

- this can be achieved indistinguishably from a standard P2PKH output
- let $\widetilde{X}$ be the MuSig aggregated key for the n parties
- output uses public key $Y = \widetilde{X} + H(\widetilde{X}, S)G$
- two ways to spend the output:
  - the n parties agree to sign with Y (one of them simply adds a corrective term $cH(\widetilde{X}, S)$ to its partial signature)
  - $\widetilde{X}$ and S are revealed and a `ScriptSig` S' is provided; the network checks $\widetilde{X} + H(\widetilde{X}, S)G \stackrel{?}{=} Y$ and that $S \| S'$ returns True

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- goal: enforce smart contracts without publishing the contract in the blockchain
- relies on "adaptor" signatures:
  - Alice has key pair $(x, X = xG)$
  - Alice draws two ephemeral keys $R = rG$, $T = tG$
  - she computes $s = r + t + H(X, R + T, m)x$ and sends (R, T, s') to Bob where $s' = s - t$
  - Bob can check $s'G \stackrel{?}{=} R + H(X, R + T, m)X$ but can't compute a valid signature for m
  - now revealing signature $s \Leftrightarrow$ revealing t
- t can be some secret value necessary for an auxiliary protocol (correctness can be proved in zero-knowledge from T)
- using a 2-of-2 multisig and an adaptor signature, one can obtain a cross-chain atomic swap protocol indistinguishable from standard spendings on each chain

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- goal: enforce smart contracts without publishing the contract in the blockchain
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  - Alice draws two ephemeral keys $R = rG$, $T = tG$
  - she computes $s = r + t + H(X, R + T, m)x$ and sends (R, T, s') to Bob where $s' = s - t$
  - Bob can check $s'G \stackrel{?}{=} R + H(X, R + T, m)X$ but can't compute a valid signature for m
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Discreet Log Contracts (T. Dryja)

- allows to enforce contracts based on external events
  - oracle Olivia has public key: pair $(X = xG, R = rG)$
  - Olivia's signature on m is simply $s_m = r + H(R, m)x$
  - for any message m, anybody can compute $S_m = s_m G = R + H(R, m)X$
- to establish a contract, Alice and Bob send funds to a shared multisig address ($\sim$ payment channels in Lightning Network)
- for each possible outcome m_i of the external event, Alice and Bob have public keys $X_{a,m_i} = x_a G + S_{m_i}$, resp. $X_{b,m_i} = x_b G + S_{m_i}$ allowing to spend from the funding channel
- when the external event happens, Olivia signs the observed outcome m_{obs}, revealing $s_{m_{\text{obs}}}$
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Outline

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Signature and Key Aggregation

Other Applications

Conclusion

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  - reduce the size of transaction and speed up verification (lower cost, less network overhead, . . .)
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The end...

Thanks for your attention!

Comments or questions?

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